KFUPM
Mathematical Sciences

Term 051
STAT 319
Quiz #6

Date: 4/1/2006
Duration: 25 minutes

Name: ID#: Serial#: Section#: 5 6

Write neatly and eligibly

Consider the following study about the relationship between the amount of nickel (the regressor) and the volume percent austenite in various steels. The table below gives the data:

X	0.608	0.634	0.651	0.658	0.675	0.677	0.702	0.71	0.73	0.75	0.772	0.802	0.819
Y	2.11	1.95	2.27	1.95	2.05	2.09	2.54	2.51	2.33	2.26	2.47	2.8	2.95

$\sum_{i=1}^{13} x_i = 9.2$, $\sum_{i=1}^{13} y_i = 30.3$, $\sum_{i=1}^{13} x_i^2 = 6.54$, $\sum_{i=1}^{13} y_i^2 = 71.71$, and $\sum_{i=1}^{13} x_i y_i = 21.61$. (1) $S_{xx} = 0.028$
$S_{yy} = 1.088$ (1) & $S_{xy} = 0.167$ (1)

Conduct all analyses at the 0.05 significance level

a. Compute the correlation coefficient.
b. Estimate the regression equation.
c. Compute the R^2 and interpret this computed value
d. Calculate the 95% confidence interval for the slope. Is the estimate any different from zero?
e. Test the intercept against the hypothesis that it is no different from zero.
f. Estimate the percent of Austenite when the amount of Nickel is 0.802
g. Calculate the 95% confidence interval for the mean percent of Austenite given the amount of Nickel is 0.802
h. Calculate the 95% prediction interval for percent of Austenite given the amount of Nickel is 0.802

2 +3 (a) $r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{\sum xy - n\bar{x}\bar{y}}{\sqrt{(\sum x^2 - n\bar{x}^2)(\sum y^2 - n\bar{y}^2)}} = \frac{21.61 - (9.2)(30.3)/13}{\sqrt{[6.54 - (9.2)^2/13][71.71 - (30.3)^2/13]}}$

$= \frac{0.167}{\sqrt{(0.028)(1.088)}}$ (1) $= \boxed{0.957}$ (1)

5 (b) $\hat{Y} = a + bX \Rightarrow b = \frac{S_{xy}}{S_{xx}} = \frac{0.167}{0.028}$ (1) $= \boxed{5.96}$ (1), $a = \bar{Y} - b\bar{X} \Rightarrow$

$a = (2.33) - (5.96)(0.707)$ (1) $= \boxed{-1.88}$ (1) $\Rightarrow \boxed{\hat{Y} = -1.88 + 5.96X}$ (1)

3 (c) $R^2 = \frac{SSR}{SST} = r^2$ (1) $= (0.957)^2 = \boxed{91.6\%}$ (1) $\Rightarrow$ That is 91.6% of the (1) variation in Y is explained by the predictor variable X.

7 (d) A $(1-\alpha)100\%$ C.I. for β is $b \pm t_{\frac{\alpha}{2}} \frac{S_e}{\sqrt{S_{xx}}}$ (1) where S_e (1) $= \sqrt{\frac{S_{yy} - bS_{xy}}{n-2}} = \sqrt{0.008} = \boxed{0.092}$ (1)

$\Rightarrow \frac{\alpha}{2} = \frac{0.05}{2} = 0.025 \Rightarrow t_{0.025, 11} = 2.201$ (1) $\Rightarrow$ A 95% C.I. for β is:

$5.96 \pm (2.201)\frac{0.092}{\sqrt{0.028}} = 5.96 \pm 1.21 = [4.75, 7.17]$ (1) $\neq 0 \Rightarrow H_0$ is rejected (1)

where $H_0: \beta = 0$ vs. $H_1: \beta \neq 0$ (1) $\Rightarrow$ The slope is definitly different from zero.

With My Best Wishes

4 (e) $H_0: \alpha = 0$ vs. $H_1: \alpha \neq 0$, $\alpha = 0.05$, $t_{0.025, 11} = 2.201$

$$t_0 = \frac{a - \alpha_0}{s\sqrt{\frac{\sum x^2}{n S_{xx}}}} \overset{(1)}{=} \frac{-1.88 - 0}{(0.092)\sqrt{\frac{6.54}{13(0.028)}}} = \frac{-1.88}{0.39} = -4.8 \quad (1)$$

Since $|t_0| = |-4.8| \overset{(1)}{<} 2.201 = t_{\frac{\alpha}{2}} \Rightarrow$ Reject H_0 (1)

$p\text{-value} = P(|t| > |t_0|) = 2P(t > 4.8) = 2(0.0005) < p\text{-value} < 2.(0.005)$

$0.001 < p\text{-value} < 0.01 \Rightarrow p\text{-value} < \alpha \Rightarrow$ Reject H_0

1 (f) $y_{0.802} = -1.88 + (5.96)(0.802) = \boxed{2.9}$ (1)

3 (g) A $(1-\alpha)100\%$ C.I. for $\mu_{y|x}$ is $\hat{y}_0 \pm t_{\frac{\alpha}{2}} S_e \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}$ (1)

$\Rightarrow$ A 95% C.I. for $\mu_{y(0.802)}$ is $2.9 \pm (2.201)(0.092)\sqrt{\frac{1}{13} + \frac{(0.802 - 0.707)^2}{0.028}}$ (1)

$= 2.9 \pm (2.201)(0.092)(0.632) = 2.9 \pm 0.128 = [2.77, 3.03]$ (1)

2 (h) A $(1-\alpha)100\%$ P.I. for $\mu_{y|x}$ is $\hat{y}_0 \pm t_{\alpha/2} S_e \sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}$ (1)

$\Rightarrow 2.9 \pm (2.201)(0.092)\sqrt{1 + \frac{1}{13} + \frac{(0.802 - 0.707)^2}{0.028}}$ (1)

$\Rightarrow 2.9 \pm (2.201)(0.092)(1.183) = 2.9 \pm 0.24 = [2.66, 3.14]$