

* SOLUTIONS * - A -

(2 pts each)

Q1. Answer the following questions by indicating it as True or False, and if it is false correct it just below to it:

F 1. In a hypothesis test, the p-value measures the probability that the alternative hypothesis is true.

① False (p-value = the smallest level of significance at which H_0 would be rejected.) ①

F 2. If a hypothesis test leads to incorrectly rejecting the null hypothesis, a Type II statistical error has been made.

① False (Type-I error) ①

T 3. It is possible to test the population proportion for categorical data type.

True

T 4. A confidence interval estimation for the difference between two population proportions $P_1 - P_2$ can be used to reject or not a two-tailed hypothesis test for $P_1 - P_2$.

True

F 5. A two-tailed hypothesis test is used when the null hypothesis looks like the following:
 $H_0: \bar{x} = 100$.

① False ($H_0: \mu = 100$) ①

1. F

2. T

3. T

4. F

5. F

(2 pts each)

Q2. Answer the following questions by choosing the right answer

1. A hypothesis test for the difference between two means is considered a two-tailed test when:
 - a. The population variances are equal.
 - b. The null hypothesis states that the population means are equal.
 - c. The alpha level is 0.10 or higher.
 - d. The p-value less than alpha
 - e. None of the above.

2. The t-distribution is used in a hypothesis test about the population mean because:
 - a. The population standard deviation is unknown and the sample size is small.
 - b. It results in a lower probability of a Type I error occurring.
 - c. It provides a smaller critical value than the standard normal distribution for a given sample size.
 - d. The population standard deviation is known or the sample size is large
 - e. None of the above.

3. A hypothesis test is to be conducted using an alpha = .05 level. This means:
 - a. There is a 5 percent chance that the null hypothesis is true.
 - b. There is a 5 percent chance that the alternative hypothesis is true.
 - c. There is a maximum 5 percent chance that a true null hypothesis will be rejected.
 - d. There is a 5 percent chance that a Type II error has been committed.
 - e. None of the above.

4. Under what conditions can the t-distribution be correctly employed to test the difference between two population means?
 - a. When the samples from the two populations are small and the population variances are unknown.
 - b. When the two populations of interest are assumed to be normally distributed.
 - c. When the population variances are assumed to be equal.
 - d. The selected samples are independent.
 - e. All of the above.

5. For testing the following null hypothesis: $H_0: P_1 - P_2 \leq 0.05$ the test statistic value given that $n_1 = 265, n_2 = 285, x_1 = 106, x_2 = 57$ equals to:

- a. 0.2964
- b. 3.8490
- c. 2.330
- d. 0.20
- e. 3.9162

$$Z_0 = \frac{(\bar{p}_1 - \bar{p}_2) - d_0}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{\left(\frac{106}{265} - \frac{57}{285}\right) - 0.05}{\sqrt{(0.296)(0.704)\left(\frac{1}{265} + \frac{1}{285}\right)}}$$

$$= 3.849$$

where $\bar{p} = \frac{106 + 57}{265 + 285} = 0.296$

a
b
c
d
e

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Q3. A company manager wishes to compare the effectiveness of two methods for training new sales people. He claims that type A training would increase the weekly sales by more than \$50 rather than type B training. To test his claim he selected 22 sales trainees who were randomly divided equally into two experimental groups – one receives type A and the other type B training. When he reviewed the performances of the salespeople in the two groups he found the following results:

	A Group	B Group	
Average Weekly Sales (in \$)	1500	1300	$n_1 = n_2 = 11$
Standard Deviation (in \$)	225	251	$\alpha = 0.10$

Using 10% level of significance, do you agree with the company manager? Explain.

The hypotheses are: $H_0: \mu_A - \mu_B \leq 50$ $H_A: \mu_A - \mu_B > 50$ (2 pts)

The assumptions are: a. The two populations are normally distributed b. Samples are small (2 pts)
 c. σ_1^2, σ_2^2 are unknown but equal d. Samples are indep.

The test statistic:

$$t_c = \frac{\bar{X}_A - \bar{X}_B - d_0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(11-1)(225)^2 + (11-1)(251)^2}{11 + 11 - 2}} = \sqrt{56813}$$

$$= 238.3548 \quad (2 \text{ pts})$$

$$\therefore t_c = \frac{1500 - 1300 - 50}{(238.3548) \sqrt{\frac{1}{11} + \frac{1}{11}}} = 1.4759 \quad (2 \text{ pts})$$

The critical value:

$$t_{\alpha, n_1+n_2-2} = t_{0.10, 20} = 1.3253 \quad (1 \text{ pt})$$

The decision rule & the decision:

Reject H_0 if $t_c > t_{\alpha, n_1+n_2-2} = 1.3253$ } (1 pt)
 Since $t_c = 1.4759 > 1.3253$
 \therefore Reject H_0 } (1 pt)

Conclusion:

Type A training WILL increase the weekly sales by more than \$50 (OR: The claim is CORRECT) (1 pt)

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Q4. Referring to the previous question and assuming that the sample of the weekly sales by type B training method has a size of 36, do you think that the average weekly sales by type B training method would differ from \$1400? Use the **p-value** approach with 5% level of significance.

$$n=36, \bar{x}=1300, s=251, \alpha=.05$$

The hypotheses are: H_0 :

$$\mu_B = 1400$$

$$H_A: \mu_B \neq 1400$$

(2) pts

The test statistic:

$$\begin{aligned} Z_c &= \frac{\bar{x}_B - \mu_0}{s_B / \sqrt{n}} \\ &= \frac{1300 - 1400}{251 / \sqrt{36}} \\ &= -2.39 \end{aligned}$$

(2) pts

The p-value:

$$\begin{aligned} P\text{-value} &= 2 P(Z > |Z_c|) \\ &= 2 P(Z > |-2.39|) \\ &= 2 P(Z > 2.39) \\ &= 2 (0.5000 - P(0 < Z < 2.39)) \\ &= 2 (0.5000 - 0.4916) \\ &= 2 (0.0084) = 0.0168 \end{aligned}$$

(2) pts

The decision:

Reject H_0 if $P\text{-value} < \alpha$
 $0.0168 < .05$
 \therefore Reject H_0

(2) pts

*** SOLUTIONS * - A -**

Using 1 for homeowners & 2 for renters

Q5. A telephone company wants to determine whether the demand on a new security system varies between homeowners and renters. Two independent random samples of 25 homeowners and 20 renters were randomly selected. It was found that 10 out of the 25 homeowners and 6 out of the 20 renters would buy the new security system. At 0.02 level of significance, do the data provide sufficient evidence to conclude that the renters are at most as interested as the homeowners in buying the new security system? Use the **p-value** approach.

Homeowners \rightarrow 1 $n_1=25$ $x_1=10$
Renters \rightarrow 2 $n_2=20$ $x_2=6$

The hypotheses are: $H_0: p_1 \geq p_2$ or $p_2 \leq p_1$ $H_A: p_1 < p_2$ or $p_2 > p_1$ (2 pts)

The assumptions are: a. $n_1 \bar{p}_1 = 25(.4) = 10 \geq 5 \checkmark$, $n_1(1-\bar{p}_1) = 25(.6) = 15 \checkmark$ (2 pts) $\bar{p}_1 = \frac{x_1}{n_1} = \frac{10}{25} = 0.40$
b. $n_2 \bar{p}_2 = (20)(.3) = 6 \geq 5 \checkmark$, $n_2(1-\bar{p}_2) = 20(.7) = 14 \geq 5 \checkmark$ $\bar{p}_2 = \frac{x_2}{n_2} = 0.30$

The test statistic value:

$$Z_c = \frac{\bar{p}_1 - \bar{p}_2 - 0}{\sqrt{\bar{p}(1-\bar{p})(\frac{1}{n_1} + \frac{1}{n_2})}}$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{10 + 6}{25 + 20} = 0.3556$$

$$= \frac{0.4 - 0.30 - 0}{\sqrt{(0.3556)(1-0.3556)(\frac{1}{25} + \frac{1}{20})}} = 0.6963$$

$$\approx 0.70$$

(2 pts)

The p-value =

$$P\text{-value} = P(Z < Z_c)$$

$$= P(Z < 0.70)$$

$$= 0.5000 + P(0 < Z < 0.70)$$

$$= 0.5000 + 0.2580$$

$$= 0.7580$$

(2 pts)

The decision rule & the decision:

Reject H_0 if the p-value $< \alpha$
 $0.7580 \not< 0.02$

\therefore Do not reject H_0

(2 pts)

The conclusion: The demand on a new security system by renters is not at most as the demand by homeowners. (1 pt)

Q6. Referring to the previous question, if $[0.208, 0.592]$ is a 95% CI on the percentage of the homeowners who would buy the new security system, do you think that the number of the homeowners who would buy the security system is the same as the number of who wouldn't buy it? Explain in detail.

$n_1 = 25, x_1 = 10$

The hypotheses are: $H_0: p = 0.5$ $H_A: p \neq 0.5$ $p_0 = q_0 = 0.5$

The decision rule: Reject H_0 if $(p_0 = 0.5)$ is outside the 95% C.I. for p .

(2) pts

OR If $p_0 \in \text{C.I.} \Rightarrow$ DO NOT reject H_0 .

The decision: $p_0 = 0.5 \in [0.208, 0.592]$

\therefore Do not reject H_0 .

(2) pts

The conclusion:

The percentage of homeowners who would buy is NOT different from the percentage of homeowners who would not buy it.

(2) pts

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OR: The two percentages are the same

OR: The percentage of the homeowners who would buy the system is 50%.