KFUPM
Mathematical Sciences

Term 051
STAT 319
Quiz #4

Date: 10/11/2005
Duration: 15 minutes

Name: ID#: Serial#: Section#: 5 6

Write neatly and eligibly

Q1 (3 points): A product designer is testing a new customer battery. A test was conducted in which the new battery was discharged in parallel with the existing model. The following results express the number of hours by which he new battery outlasted the old one:

5 -4 10 15 11 25 -5 17 -5 0 8 12 4 8 1 5

Compute the maximum estimation error with a 99% confidence level for estimating the mean life time advantage of the new battery, and interpret the result.

$n = 16, \ \bar{x} = 6.69, \ s^2 = 69.96, \ s = 8.36$

$e = t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}} = t_{0.005, 15} \frac{s}{\sqrt{n}} = \frac{(2.947)(8.36)}{4} = \boxed{6.159}$

This means that the difference between the limits of any 99% C.I. about μ will not exceed $2(6.159) = 12.32$ hrs.

Q2 (3 points): An automobile parts distributor found 34 packages containing defective brake linings in a sample of 200 taken at random from a shipment containing 1,000 packages. Find the minimum number of packages that must be drawn from the 1,000 packages to estimate the proportion of defective packages with a 98% level of confidence and with a maximum estimation error of 0.04.

$n = 200, \ x = 34 \Rightarrow \hat{p} = \frac{34}{200} = \boxed{0.17}, \ \hat{q} = 0.83, \ e = 0.04$

$1 - \alpha = 0.98 \Rightarrow \alpha = 0.02 \Rightarrow \frac{\alpha}{2} = 0.01 \Rightarrow z_{\frac{\alpha}{2}} = z_{0.01} = \boxed{2.33}$

$n \geq \frac{z^2_{\alpha/2} \hat{p}\hat{q}}{e^2} = \frac{(2.33)^2 (0.17)(0.83)}{(0.04)^2} = 478.76 \approx \boxed{479}$ packages

Q3 (4 points): Referring to Q1, construct a 95% C.I. for the standard deviation of the life time advantage of the new battery, and interpret the result.

A $(1-\alpha)100\%$ C.I. for σ^2 is $\left[\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}}\right] \Rightarrow$

$1 - \alpha = 0.95 \Rightarrow \frac{\alpha}{2} = \frac{0.05}{2} = 0.025, \ n - 1 = 15$

$\Rightarrow \chi^2_{\frac{\alpha}{2}, n-1} = \chi^2_{0.025, 15} = \boxed{27.49}$ & $\chi^2_{1-\frac{\alpha}{2}, n-1} = \chi^2_{0.975, 15} = \boxed{6.26}$

$\Rightarrow$ A 95% C.I. for σ^2 = $\left[\frac{15(69.96)}{27.49}, \frac{15(69.96)}{6.26}\right] = [38.174, 167.636]$

$\Rightarrow$ A 95% C.I. for σ = $[\sqrt{38.174}, \sqrt{167.636}] = [6.18, 12.95]$

We are 95% Confident that the standard deviation of the population falls in

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