KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
DHAHRAN, SAUDI ARABIA

STAT 319: PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS

Major Exam No. II, Fall Semester (051)
Time: 0900 - 1000 pm, Tuesday 29th November, 2005

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Serial # Surname: ID# Section #

Circle the **name** of the **instructor**. You are allowed to use electronic calculators and other reasonable accessories that help write the exam. Try to define events, formulate problem and solve.

1. Turn off your mobile and leave it aside.
2. **Write** your name, ID number, Section, and serial number (optional) on the cover page.
3. You can **only** ask **questions** if you feel there is any mistake in the exam needing announcement for correction.

Q	Full Marks	Marks Obtained	Strengths/ Weakness
1	5		
2	5+~~3~~ 5+5=~~13~~ 15		
3	5		
4	5		
5	3+2+5=10		
6	5+~~3~~ 5=~~8~~ 10		
Total	~~46~~ 50		

other 9 for a larger project. Suppose that 5 of the 15 have a crushing strength below the specified minimum. If the 6 for the smaller project are randomly selected from the 15, then find the probability that at least one of of them have crushing strength below the specified minimum. **[5 Marks]**

Let X: # of cylinders having strength below min.

$N = 15,\ n = 6,\ k = 5,\ X: HG(15, 6, 5)$

$P(X \geq 1) = 1 - P(X = 0)$

$= 1 - \dfrac{\binom{5}{0}\binom{10}{6}}{\binom{15}{6}} = 1 - \dfrac{210}{5005}$

$= 1 - 0.042 = \boxed{0.958}$

2. Suppose that 10% of all steel shafts produced by a certain process are nonconforming but can be reworked (rather than having to be scrapped).
a. Find the probability that 3 of the 5 shafts selected are nonconforming.
b. Find the probability that the first nonconforming shaft was found in the 4th trial.
b. The ***approximate*** probability that less than 30 nonconforming shafts were found in a sample of 200 shafts. [5+5+5=15 Marks

$p = 0.1$

a. $n = 5$, X: # of nonconforming shafts.

$X: B(5, 0.1) \Rightarrow f(x) = \binom{5}{x}(0.1)^x(0.9)^{5-x},\ x = 0, 1, \dots, 5$

$P(X = 3) = \binom{5}{3}(0.1)^3(0.9)^2 = 10(0.001)(0.81)$

$= \boxed{0.0081}$

b. Let X: # of trials on which nonconforming shaft is found first.

$X: G(0.1) \Rightarrow f(x) = (0.1)(0.9)^{x-1},\ x = 1, 2, \dots$

$P(X = 4) = (0.1)(0.9)^3 = \boxed{0.075}$

c. $n = 200,\ p = 0.1 \Rightarrow \boxed{np = 20 \geq 5}\ \boxed{nq = 180 \geq 5}$

Let X: # of shafts in a sample of 200.

$X \sim N(20, 18) \Rightarrow \mu = 20,\ \sigma = 3\sqrt{2}$

$P(X < 30) = P(X \leq 29) = P(X \leq 29.5)$

$= P\left(Z \leq \dfrac{29.5 - 20}{3\sqrt{2}}\right) = P(Z \leq 2.24)$

$= \boxed{0.9875}$

standard deviation 10 hours. If a randomly selected pack has **40** batteries, then what is the probability that the mean lifetime of this pack will exceed 11 hours? **[5 Marks]**

Let X: Life time of a battery. $n = 40$ (Large)

By CLT $\bar{X} \sim N(10, \frac{100}{40})$, $\mu_{\bar{x}} = 10$, $\sigma_{\bar{x}} = 1.581$

$$P(\bar{X} > 11) = P\left(Z > \frac{11-10}{1.581}\right) =$$

$$= P(Z > 0.63) =$$

$$= P(Z < -0.63) = \boxed{0.2643}$$

4. The time (Y) at the post office in Riyad that you have to wait until you are served has an **exponential distribution** with mean 12 minutes. The pdf of the exponential distribution is given by $f(y) = (1/\beta)\, e^{-y/\beta},\ 0 < y;\ f(y) = 0$ elsewhere. Find the probability that you have to wait at least 15 minutes until you are served at the post office of Riyad. [5 Marks]

If $Y : Exp(\beta) \Rightarrow F(y) = P(Y \le y) = \int_0^y f(t)\,dt$

$$F(y) = \int_0^y \frac{e^{-t/\beta}}{\beta}\,dt = -e^{-t/\beta}\Big]_0^y$$

$$= 1 - e^{-t/\beta}, \quad \beta = 12$$

$$P(Y \ge 15) = 1 - P(Y < 15) = 1 - F(15)$$

$$= 1 - (1 - e^{-15/12}) = e^{-15/12} =$$

$$= \boxed{0.2865}$$

with mean 12.5 mm^2 and standard deviation 0.2 mm^2. When the area is less than 12 mm^2 or greater than 13 mm^2, the tube does not fit properly.

(a) What proportion of the tubes fit properly?

(b) If the tubes are shipped in boxes of 1000, how many wrong sized tubes per box can doctors expect to find?

(c) A sample of size 9 tubes is selected, what is the probability that sample mean will be more than 12.5 mm^2? [3+2+5=10 Marks]

X: Cross sectional area, $\mu = 12.5$, $\sigma = 0.2$

$\Rightarrow X : N(12.5, 0.04)$

a. $P(\text{Fitting}) = P(12 < X < 13) = P\left(\frac{12-12.5}{0.2} < Z < \frac{13-12.5}{0.2}\right)$ (1)

$= P(-2.5 < Z < 2.5) =$

$= P(Z < 2.5) - P(Z < -2.5)$ (1)

$= 0.9938 - 0.0062 = \boxed{0.9876}$ (1)

b. $P(\text{wrong size}) = 1 - 0.9876 = 0.0124$ (1)

$\Rightarrow$ # wrong sized in a box $= 1000(0.0124)$

$= 12.4 \approx 12$ tubes (1)

c. $n = 9 \Rightarrow \bar{X} : N\left(\mu, \frac{\sigma^2}{n}\right) \Rightarrow$ (1)

$\mu_{\bar{x}} = 12.5$, (1) $\sigma^2_{\bar{x}} = \frac{0.04}{9} \Rightarrow \sigma_{\bar{x}} = \frac{0.2}{3} = 0.067$

$P(\bar{X} > 12.5) = P\left(Z > \frac{12.5 - 12.5}{0.067}\right) = \boxed{0.5}$

(1) (1) (1)

6.A bank manager wishes to provide prompt service for customers at the bank window. The average arrival rate is 7 customers per 15 minute period. Find the probability that 15 customers will arrive in 30 minute period.

a) The bank currently serve up to 10 customers per 15 minute period without significant delay.

b) Find the probability that there will be a significant delay at the window in next 15 minutes. [5+~~3~~5 =~~8~~ Marks]

X: # of Customers in 15 min. $\Rightarrow X : Po(\lambda)$, $\lambda = 7/15$ min (1) (1)

(1) $f(x) = \frac{e^{-7} 7^x}{x!}$, $x = 0, 1, 2, \ldots$

a. $\lambda = 7$, $t = 2$ (1) $\Rightarrow \lambda t = 14$ (1) $\Rightarrow P(X = 15) = \frac{e^{-14} 14^{15}}{15!}$ (1) $= \boxed{0.099}$ (1)

b. $P(X > 10) = 1 - P(X \le 10) = 1 - \sum_{x=0}^{10} \frac{e^{-7} 7^x}{x!}$

(1) (1) (1)