

## CHAPTER SEVEN

### Tests of Hypotheses

#### 7.1 Testing Hypotheses about a Population Mean

##### Testing Hypotheses on the Mean of a Normal Population, Variance Known

Possible hypotheses, rejection region and probability values are summarized in the following table below.

**Table 1** Testing hypotheses about a population mean using  $z$  tests

$H_0$ vs. $H_a$	Rejection Region	$p$ -value
$\mu = \mu_0$ vs. $\mu < \mu_0$	$z < -z_\alpha$	$P(Z < z)$
$\mu = \mu_0$ vs. $\mu > \mu_0$	$z > z_\alpha$	$P(Z > z)$
$\mu = \mu_0$ vs. $\mu \neq \mu_0$	$z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$	$2 P(Z >  z )$

where  $z$  is the test statistic, which can be written

$$z = \frac{\bar{x} - \mu_0}{\sqrt{\sigma^2 / n}}$$

and  $\alpha$  is the significance level of the test.

**Example 7.1** The average zinc concentration recovered from a sample of zinc measurements in 36 different locations is found to be 2.6 grams per millimeter. Assume that the population standard deviation is 0.3. It is believed that the average zinc concentration of such measurements is less than 3 grams per millimeter. Set up suitable hypotheses and test at 1% level of significance.

**Solution** From the sample we have  $n = 36$ ,  $\bar{x} = 2.6$ . The hypotheses are given by

$$H_0 : \mu = 3 \quad \text{vs} \quad H_a : \mu < 3$$

The value of the test statistic  $z$  is given by

$$z = \frac{2.6 - 3}{\sqrt{(0.3)^2 / 36}} = -8$$

Since  $z = -8 < -z_\alpha = -2.33$  we reject the null hypothesis  $H_0$ , i.e. there is sufficient evidence to reject the hypothesis that the mean zinc concentration is 3.

##### Large Sample Test of the Population Mean

Refer to Table 1 for the hypotheses. The test statistic is given by

$$z = \frac{\bar{x} - \mu_0}{\sqrt{s^2 / n}}$$

**Example 7.2** A manufacturer of sports equipment has developed a new synthetic fishing line that he claims has a mean breaking strength of 8 kilograms. At 1% level of significance, test the hypothesis that the mean breaking strength is 8 kilograms against the alternative that mean breaking strength is not 8 kilograms if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kilograms and a standard deviation of 0.5 kilogram.

**Solution** The hypotheses are given by

$$H_0 : \mu = 8 \quad \text{vs} \quad H_a : \mu \neq 8$$

The value of the test statistic  $z$  is given by

$$z = \frac{7.8 - 8}{\sqrt{(0.5)^2 / 50}} = -2.83$$

Since  $z = -2.83 < -z_{0.005} = -2.575$  we reject  $H_0$  and conclude that the average breaking strength is not equal to 8.

### Testing the Mean of a Normal Population, Variance Unknown

**Table 2** Testing hypotheses about a population mean using  $t$  tests

$H_0$ vs. $H_a$	Rejection Region	$p$ -value
$\mu = \mu_0$ vs. $\mu < \mu_0$	$t < -t_\alpha$	$P(T < t)$
$\mu = \mu_0$ vs. $\mu > \mu_0$	$t > t_\alpha$	$P(T > t)$
$\mu = \mu_0$ vs. $\mu \neq \mu_0$	$t < -t_{\alpha/2}$ or $t > t_{\alpha/2}$	$2 P(T >  t )$

The test statistics  $t$  is given by

$$t = \frac{\bar{x} - \mu_0}{\sqrt{s^2 / n}}$$

and  $t_\alpha$  is the  $100(1-\alpha)^{\text{th}}$  percentile of student  $t$  distribution with  $n-1$  degrees of freedom.

**Example 7.3** It is claimed that a vacuum cleaner expends an average of 46 kilowatt-hours per year. If a random sample of 12 homes included in a planned study indicates that vacuum cleaners expend the following kilowatt-hours per year

30 44 40 45 46 40 47 48 46 45 41 50

Does this suggest at the 5% level of significance that vacuum cleaners expenses, on the average, is different from 46 kilowatt-hours annually? Assume that the population of kilowatt-hours to be normal.

**Solution** The hypotheses are given by

$$H_0 : \mu = 46 \quad \text{vs} \quad H_a : \mu \neq 46$$

The value of the test statistic  $t$  is given by

$$t = \frac{43.5 - 46}{\sqrt{(5.265669)^2 / 12}} = -1.6447$$

Since  $t = -1.6447$  is not less than  $t_{\alpha/2} = t_{.025} = -2.201$ , we can not reject the null hypothesis, i.e.  $\mu = 46$ .

To solve the problem using Statistica, we follow the steps:

1. Statistics
2. Basic Statistics / Tables
3. T-test, single sample / OK
4. In T-test for Single Means: Spreadsheet, Click Advanced to get Figure 7.1
5. Variables( select the variable say Var1) / OK
6. In Reference values input the value of  $\mu_0$
7. Summary

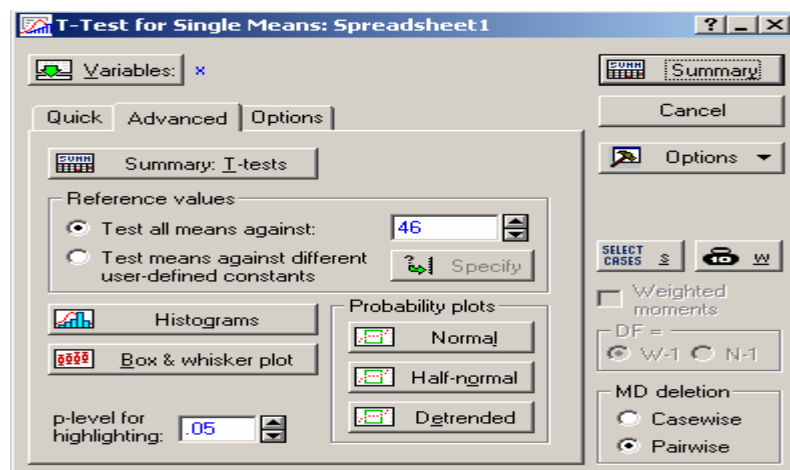


Figure 7.1 T-test for Single Means

These steps will give the following results ( Figure 7.2):

Variable	Mean	Std.Dv.	N	Std.Err.	Reference Constant	t-value	df	p
x	43.50000	5.265669	12	1.520068	46.00000	-1.64466	11	0.128285

Figure 7.2 Test of means against reference constant (value)

Since the  $p$ -value is too large compared to  $\alpha = 0.05$ , we cannot reject the null hypothesis at the 5% level of significance.

## 7.2 Testing the Difference between Two Population Means

### Testing the Difference between the Means of Two Independent Normal Populations, Known Variances

**Table 3** Testing hypotheses about the difference between the means of two populations using  $z$  tests

$H_0$ vs. $H_a$	Rejection Region	$p$ -value
$\mu_1 - \mu_2 = d_0$ vs. $\mu_1 - \mu_2 < d_0$	$z < -z_\alpha$	$P(Z < z)$
$\mu_1 - \mu_2 = d_0$ vs. $\mu_1 - \mu_2 > d_0$	$z > z_\alpha$	$P(Z > z)$
$\mu_1 - \mu_2 = d_0$ vs. $\mu_1 - \mu_2 \neq d_0$	$z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$	$2 P(Z >  z )$

Since the assumption of known variances are not that realistic, we do not consider any example here.

### Large Sample Test of the Means of Two Independent Populations, Unknown Variances

The test statistic  $z$  is given by

$$z = \frac{\bar{x}_1 - \bar{x}_2 - d_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

**Example 7.4** Consider a tire manufacturer who wishes to estimate the difference between the mean lives of two types of tires, Type A and Type B, as a prelude to a major advertising campaign. A sample of 100 tires is taken from each production process. The sample mean lifetimes are 30100 and 25200 miles respectively; the sample variances are 1500000 and 2400000 miles squared respectively. Is there any difference between the mean lives of the two types of tires at the 1% level of significance?

**Solution:** The hypotheses are given by

$$H_o : \mu_1 = \mu_2 \text{ (i.e. } \mu_1 - \mu_2 = 0) \quad \text{vs} \quad H_a : \mu_1 \neq \mu_2 \text{ (i.e. } \mu_1 - \mu_2 \neq 0)$$

The value of the test statistic  $z$  is given by

$$z = \frac{30100 - 25200 - 0}{\sqrt{\frac{1500000}{100} + \frac{2400000}{100}}} = 24.812$$

Since  $z = 24.812 > z_{\alpha/2} = 2.575$ , we reject  $H_o$  and accept  $H_a$ .

## Testing the Difference between the Means of Two Independent Normal Populations, Unknown But Equal Variances

**Table 4** Testing hypotheses about the difference between the means of two populations using  $t$  tests

$H_0$ vs. $H_a$	Rejection Region	$p$ -value
$\mu_1 - \mu_2 = d_0$ vs. $\mu_1 - \mu_2 < d_0$	$t < -t_\alpha$	$P(T < t)$
$\mu_1 - \mu_2 = d_0$ vs. $\mu_1 - \mu_2 > d_0$	$t > t_\alpha$	$P(T > t)$
$\mu_1 - \mu_2 = d_0$ vs. $\mu_1 - \mu_2 \neq d_0$	$t < -t_{\alpha/2}$ or $t > t_{\alpha/2}$	$2 P(T >  t )$

The test statistic  $t$  is given by

$$t = \frac{\bar{x}_1 - \bar{x}_2 - d_0}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

with  $\nu = (n_1 - 1) + (n_2 - 1)$  degrees of freedom and the pooled variance is given by

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}.$$

**Example 7.5** A random sample of 15 bulbs produced by an old machine was tested and found to have a mean life span of 40 hours with standard deviation 5 hours. Also, a random sample of 10 bulbs produced by a new machine was found to have a mean life span of 45 hours with standard deviation 6 hours. Assume that the life span of a bulb has a normal distribution for both machines, and true variances are the same. Test at the 5% level of significance if there is any difference between the mean lives of the bulbs produced by two machines.

**Solution** The hypotheses are given by

$$H_o : \mu_1 - \mu_2 = 0 \quad \text{vs} \quad H_a : \mu_1 - \mu_2 \neq 0$$

Here,  $n_1 = 15$ ,  $\bar{x}_1 = 40$ ,  $s_1^2 = 25$ ,  $n_2 = 10$ ,  $\bar{x}_2 = 45$ ,  $s_2^2 = 36$ ,  $\nu = (n_1 - 1) + (n_2 - 1) = 23$  and  $\alpha = 0.05$ . The estimate of common variance  $\sigma^2$  is  $s_p^2 = 29.304$ . The value of  $t$  given by

$$t = \frac{(40 - 45) - 0}{(5.4166)\sqrt{\frac{1}{15} + \frac{1}{10}}} = -2.2625$$

Since  $t = -2.2625 < -t_{\alpha/2} = -2.069$ , we reject the null hypothesis.

To solve the above problem using Statistica, the following steps can be used

1. Statistics
2. Basic Statistics / Tables
3. Difference tests r, %, means / OK

4. In Difference between two mean (Normal distribution), input the values of M1, M2, StDv1, StDv2, N1 and N2, then press Compute to get Figure 7.3.

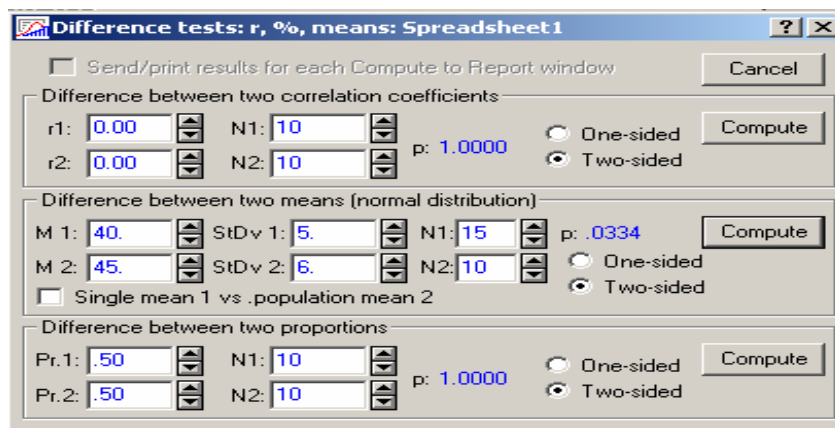


Figure 7.3 Difference tests

The  $p$ -value = 0.0334 (See Figure 7.3) provided by Difference Tests in Statistica is very much in agreement with what we got by the probability calculator (in Statistica). Since the  $p$ -value = 0.0334 < 0.05, we reject the null hypothesis at the 5% level of significance. Note that the option of one sided alternative hypotheses is available in Statistica (See Figure 7.3).

**Example 7.6** In order to compare two lifting clubs, Club A and Club B, a sample of twenty weight liftings from the same division were studied resulting in the data below:

Club A	251	247	249	251	249	250	254	245	257	249
	251	254	259	250	252	256	243	251	242	248
Club B	249	251	259	248	259	252	249	251	251	253
	250	249	251	250	253	257	249	253	249	251

We want to test the following hypotheses:

$$H_o : \mu_A = \mu_B \text{ (the population means are identical)}$$

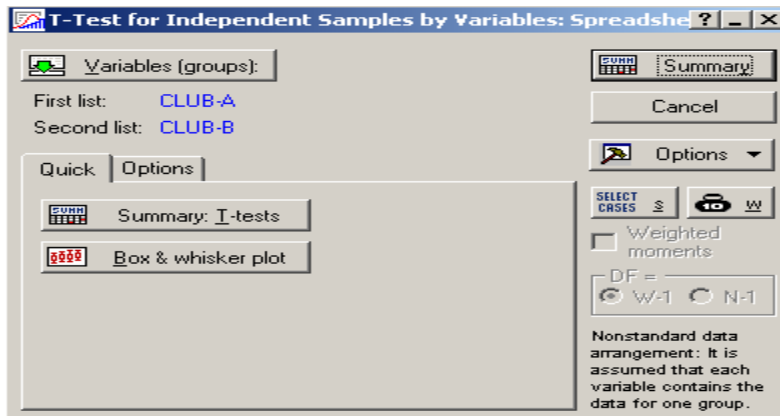
$$H_a : \mu_A \neq \mu_B \text{ (the population means are not identical)}$$

The sample mean and variance for Club A are  $\bar{x}_1 = 250.4$  and  $s_1^2 = 18.7789$ , and the sample mean and variance for Club B are  $\bar{x}_2 = 251.7$  and  $s_2^2 = 10.4316$ . The pooled variance  $s_p^2 = 14.6053$  so that the value of test statistic is  $t = -1.0757$ , with  $df = \nu = (n_1 - 1) + (n_2 - 1) = 38$

Since the observed  $t = -1.0757$  is not less than  $-t_{\alpha/2} = -2.021$ , we accept the null hypothesis.

To solve the above problem by Statistica, we need the following steps:

1. Enter each sample in a separate column
2. Statistics / Basic Statistics and Tables
3. t-test, independent, by variables (see Figure 7.4) / OK



**Figure 7.4** T-test for Independent Variables

4. Click Variables [groups] and select CLUB-A for first list and CLUB-B for second list / OK
5. In Quick click Summary: T-tests. These steps give a scroll sheet of results (Figure 7.5).

T-test for Independent Samples (Spreadsheet1)											
Note: Variables were treated as independent samples											
Group 1 vs. Group 2	Mean	Mean	t-value	df	p	Valid N	Valid N	Std.Dev.	Std.Dev.	F-ratio	p
	Group 1	Group 2				Group 1	Group 2	Group 1	Group 2	Variances	Variances
CLUB-A vs. CLUB-B	250.4000	251.7000	-1.07569	38	0.288847	20	20	4.333468	3.229795	1.800202	0.2092

**Figure 7.5** Results of t-test

Since the  $p$ -value = 0.28847  $\gg$  0.05, we cannot reject the null hypothesis at 5% level of significance.

### Testing the Difference between the Means of Two Independent Normal Populations, Unknown Variances and Equal Sample Sizes

Refer to Table 4 for hypotheses. The value of the test statistic  $t$  is given by

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{n}}}$$

with  $2(n-1)$  degrees of freedom.

**Example 7.7** A random sample of 16 bulbs produced by an old machine was tested and found to have a mean life span of 40 hours with standard deviation 5 hours. Also, another random sample of 16 bulbs produced by a new machine was found to have a mean life span of 45 hours with standard deviation 6 hours. Assume that the life span of a bulb has a normal distribution for both machines, test at the 1% level of significance if the mean life of the bulbs produced by the new machine is more than that by the old machine.

**Solution** The hypotheses to be tested are given by

$$H_o : \mu_1 = \mu_2 \text{ (i.e. } \mu_1 - \mu_2 = 0) \quad \text{vs} \quad H_a : \mu_1 < \mu_2 \text{ (i.e. } \mu_1 - \mu_2 < 0)$$

Since there is no information about the equality of variances but the sample sizes are the same we would go for the simpler test given by

$$t = \frac{(40 - 45) - 0}{\sqrt{\frac{25}{16} + \frac{36}{16}}} \approx -2.561$$

with  $\nu = (n - 1) + (n - 1) = 15 + 15 = 30$  df. With  $\alpha = 0.01$ , since  $t = -2.561 < -t_\alpha = -2.457$ , we reject the null hypothesis.

### Testing the Difference between Means of Two Normal Populations, Neither the Variances nor the Sample Sizes are Equal

Refer to Table 4 for the hypotheses testing. The value of the test statistic  $t$  is given by

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

with degrees of freedom  $\nu = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$

**Example 7.8** A random sample of 12 bulbs produced by an old machine was tested and found to have a mean life span of 40 hours with variance 24 hours. Also, a random sample of 10 bulbs produced by a new machine was found to have a life span of 45 hours with variance 30 hours. Assume that the life span of a bulb has a normal distribution for both machines, and but the true variances are not the same. Test at the 5% level of significance if there is any difference between the mean lives of the bulbs produced by two machines.

**Solution** The hypotheses to be tested are given by

$$H_o : \mu_1 - \mu_2 = 0 \quad \text{vs} \quad H_a : \mu_1 - \mu_2 \neq 0.$$

For old machine, we have  $\bar{x}_1 = 40$ ,  $s_1^2 = 24$ ,  $n_1 = 12$  and for new machine, we have  $\bar{x}_2 = 45$ ,  $s_2^2 = 30$ ,  $n_2 = 10$ . The degrees of freedom is  $\nu = 18.3 \approx 18$  and the test



statistic is  $t = -2.236$ . Since  $t = -2.236 < -t_{\alpha/2} = -2.101$ , we reject the null hypothesis at the 5% level of significance.

### Testing the Difference between Two Population Means; Matched Pairs Case

Refer to Table 4 for the hypotheses. The test statistic  $t$  is given by

$$t = \frac{\bar{d}}{s_d / n}$$

where  $\bar{d}$  is the mean of the differences between the paired observations, and  $s_d$  is the corresponding standard deviation, and the number of degrees of freedom is  $(n - 1)$ .

**Example 7.9** For the data in Example 6.9, test at 5% level of significance if there is any difference in mean time by CAD and by Traditional Method.

**Solution** We want to test the hypotheses that

$$H_o : \mu_1 - \mu_2 = 0 \quad \text{vs} \quad H_a : \mu_1 - \mu_2 \neq 0$$

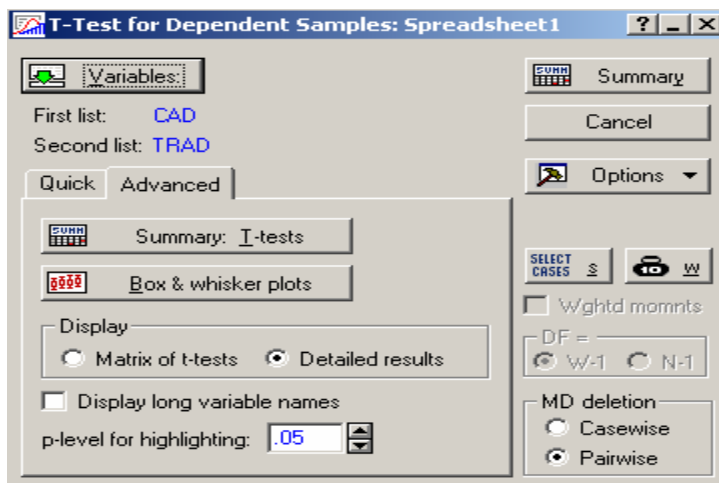
The value of the test statistic is

$$t = \frac{-2.19}{\sqrt{9.5854/10}} = -2.237$$

With  $\alpha = 5\%$  and  $df = 9$ ,  $t_{0.025} = -2.262$ . Since  $t = -2.237$  is not less than  $t_{0.025} = -2.262$ , we do not reject the null hypothesis.

The above problem can be solved using Statistica following the steps:

1. Enter each sample in a separate column
2. Statistics / Basic Statistics and Tables
3. t-test, Dependent sample (see Figure 7.6) / Ok
4. Variables (First list and Second list) / OK
5. In Quick: click **Summary: T-test**



**Figure 7.6** T-test for Dependent Samples

Since the  $p$ -value = 0.052110 (See Figure 7.7), we reject the null hypothesis at  $\alpha = 0.06$  (in fact for any  $\alpha \geq p$ -value) but accept at  $\alpha = 0.05$  (in fact for any  $\alpha < p$ -value). Note that many statisticians support keeping  $\alpha$  at a low level not exceeding 5%.

Variable	Mean	Std.Dv.	N	Diff.	Std.Dv. Diff.	t	df	p
CAD	9.96000	4.878798						
TRAD	12.15000	6.614336	10	-2.19000	3.096037	-2.23686	9	0.052110

Figure 7.7 Spreadsheet for t-test (Dependent Samples)

## 7.3 Large Sample Tests of Proportions

### Testing a Population Proportion

Tests on  $p$  are summarized in the following table:

$H_0$ vs. $H_a$	Rejection Region	$p$ -value
$p = p_0$ vs. $p < p_0$	$z < -z_\alpha$	$P(Z < z)$
$p = p_0$ vs. $p > p_0$	$z > z_\alpha$	$P(Z > z)$
$p = p_0$ vs. $p \neq p_0$	$z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$	$2 P(Z >  z )$

The test statistics  $z$  is given by

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

**Example 7.10** In certain water-quality studies, it is important to check for the presence or absence of various types of microorganisms. Suppose 20 out of 100 randomly selected samples of a fixed volume show the presence of a particular microorganism. At the 1% level of significance, test the hypothesis that the true proportion of the presence of a particular microorganism is at least 0.30.

**Solution** The hypotheses are given by

$$H_o : p = 0.30 \quad \text{vs} \quad H_a : p > 0.30.$$

with  $\alpha = 0.01$ , the Rejection Region is given by. Since the observed

$$z = \frac{0.20 - 0.30}{\sqrt{0.30(0.70)/100}} = -2.18$$

Since  $z = -2.18$  is not less than  $z_{0.005} = 2.33$ , we accept the null hypothesis.

## Large Sample Test of the Difference between Two Population Proportions

**Table 6** Testing hypotheses about the difference between two population proportions

$H_0$ vs. $H_a$	Rejection Region	$p$ -value
$p_1 - p_2 = d_1$ vs. $p_1 - p_2 < d_1$	$z < -z_\alpha$	$P(Z < z)$
$p_1 - p_2 = d_1$ vs. $p_1 - p_2 > d_1$	$z > z_\alpha$	$P(Z > z)$
$p_1 - p_2 = d_1$ vs. $p_1 - p_2 \neq d_1$	$z < -z_{\alpha/2}$ or $z > z_{\alpha/2}$	$2 P(Z >  z )$

where the test statistic  $z$  is given by

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}}, \text{ where } \hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$$

Note that  $x_1$  is the number of items in the sample from Population I having a particular characteristic, and  $x_2$  is the number of items in the sample from Population II having a particular characteristic

**Example 7.11** The fraction defective product produced by two production lines is being analyzed. A random sample of 1000 units from Line I has 10 defectives, while a random sample of 1200 units from Line II has 25 defectives. Is there any significant difference between the fraction defectives of the two production lines? (Assume that  $\alpha = 0.01$ )

**Solution** The hypotheses are given by

$$H_0 : p_1 = p_2 \quad \text{vs} \quad H_a : p_1 \neq p_2.$$

Here  $\hat{p} = \frac{10 + 25}{1000 + 1200} = \frac{35}{2200} = 0.01591$ . The value of the test statistic is given by

$$z = \frac{(10/1000 - 25/1200) - (0)}{\sqrt{\frac{(35/2200)(2165/2200)}{1000} + \frac{(35/2200)(2165/2200)}{1200}}} = -2.0221.$$

Since  $z = -2.0221$  is not less than  $z_{.005} = -2.575$ , we accept the null hypothesis, and conclude that the evidence does not support the difference.

To solve the above problem by using Statistica, we need to the following steps

1. Statistics
2. Basic Statistics / Tables
3. Difference tests r, %, means / OK to get Figure 7.8

**Difference tests: r, %, means: Spreadsheet1**

Send/print results for each Compute to Report window Cancel

**Difference between two correlation coefficients**

r1: 0.00 N1: 10 p: 1.0000  One-sided Compute  
 r2: 0.00 N2: 10  Two-sided

**Difference between two means (normal distribution)**

M 1: 1. StDv 1: .1 N1: 10 p: 1.0000 Compute  
 M 2: 1. StDv 2: .1 N2: 10  One-sided  
 Single mean 1 vs .population mean 2  Two-sided

**Difference between two proportions**

Pr.1: .01 N1: 1000 p: .0433  One-sided Compute  
 Pr.2: .02 N2: 1200  Two-sided

**Figure 7.8** Difference tests

We put  $\hat{p}_1 = 0.01$  for Pr.1 and  $\hat{p}_2 \approx 0.02$  for Pr.2 in Figure 7.8. Since the  $p$ -value = 0.0433 < 0.05, we reject the null hypothesis at 5% level of significance.

## Exercises

- 7.1 (cf. Johnson, R. A., 2000, 126). The following random samples are measurements of the heat-producing capacity (in millions of calories per ton) of specimens of coal from two mines:

Mine 1	8260	8130	8350	8070	8340	7990
Mine 2	7950	7890	7900	8140	7920	7840

- (a) Use the 0.01 level of significance to test whether the difference between the means of these two samples is significant, assuming equal variances.  
 (b) Repeat part (a) (assuming unequal variances).
- 7.2 (Johnson, R. A., 2000, 264). The following are the average weekly losses of worker-hours due to accidents in 10 industrial plants before and after a certain safety program was put into operation:

Before	45	73	46	124	33	57	83	34	26	17
After	36	60	44	119	35	51	77	29	24	11

Use the 0.05 level of significance to test whether the safety program is effective.

- 7.3 (cf. Johnson, R. A., 2000, 266). As part of an industrial training program, some trainees are instructed by Method A, which is straight teaching machine instruction and some are instructed by Method B, which also involve the personal attention of an instructor. If random samples of size 10 are taken from large group of trainees instructed by each of these two methods, and the scores which they obtained in an appropriate achievement test are

Method A	71	75	65	69	73	66	68	71	74	68
Method B	72	77	84	78	69	70	77	73	65	75

- (a) Use the 0.05 level of significance to test the claim that both the methods have same results. Assume that  $\sigma_1^2 = \sigma_2^2$ .  
 (b) Use the 0.05 level of significance to test the claim that both the methods have same results. Assume that  $\sigma_1^2 \neq \sigma_2^2$ .
- 7.4 (cf. Johnson, R. A., 2000, 266). The following are the number of sales which a sample of 9 salespeople of industrial chemicals in California and a sample of 6 salespeople of industrial chemicals in Oregon made over a certain fixed period of time:

California	59	68	44	71	63	46	69	54	48
Oregon	50	36	62	52	70	41	58	39	60

Test the null hypothesis  $\mu_1 - \mu_2 = 0$  against the alternative hypothesis  $\mu_1 - \mu_2 \neq 0$  at the 0.01 level of significance.

- 7.5 (Johnson, R. A., 2000, 266). The following are the Brinell hardness values obtained by samples of two magnesium alloys:

Alloy 1	66.3	63.5	64.9	61.8	64.3	64.7	65.1	64.5	68.4	63.2
Alloy 2	71.3	60.4	62.4	63.9	68.8	70.1	64.8	68.9	65.8	66.2

Use the 0.05 level of significance to test the null hypothesis  $\mu_1 - \mu_2 = 0$  against the alternative hypothesis  $\mu_1 - \mu_2 \neq 0$ .

- 7.6 (Johnson, R. A., 2000, 267). To compare two kinds of bumper guards, 6 of each kind were mounted on a certain kind of compact car. Then each car was run into a concrete wall at 5 miles per hour, and the following are the costs of the repairs (in dollars):

Bumper Guard 1	107	148	123	165	102	119
Bumper Guard 2	134	125	112	151	133	129

Use the 0.01 level of significance to test whether there is a difference between the two population means.

- 7.7 (Johnson, R. A., 2000, 267). The following data were obtained in an experiment designed to check whether there is a systematic difference in the weights obtained with two different scales:

Rock	1	2	3	4	5	6	7	8	9	10
Scale 1	11.23	14.36	8.33	10.50	23.42	9.15	13.47	6.47	12.40	19.38
Scale 2	11.27	14.41	8.35	10.52	23.41	9.17	13.52	6.46	12.45	19.35

Use the 0.05 level of significance to test whether the difference of the weights obtained with the two scales is significant.

- 7.8 (cf. Montgomery, D. C., et. al, 2001, 229). Two catalysts are being analyzed to determine how they affect the mean yield of a chemical process. A test is run in the pilot plant and results are;

Catalyst 1	91.50	94.18	92.18	95.39	91.79	89.07	94.72	89.21
Catalyst 2	89.19	90.95	90.46	93.21	97.19	97.04	91.07	92.75

Is there any difference between the mean yields? Use  $\alpha = 0.05$ .

- (a) Assuming equal population variances.  
 (b) Assuming unequal population variances.

- 7.9 (Johnson, R. A., 2000, 267). In a study of the effectiveness of physical exercise in weight reduction, a group of 14 persons engaged in a prescribed program of physical exercise for one month showed the following results:

Weights before (Ibs)	209	178	169	212	180	192	180
	196	171	170	207	177	190	180
Weights after (Ibs)	170	153	183	165	201	179	144
	164	152	179	162	199	173	140

Use the 0.01 level of significance to test whether the prescribed program of exercise is effective. Montgomery, Runger and Hubele (2001).

- 7.10 (cf. Montgomery, D. C., et al, 2001, 224). Two machines M1 and M2 are used for filling plastic bottles with a net volume of 16.0 ounces. A member of quality engineering staff suspects that both machines fill to the same mean net volume. A random sample of 10 bottles is taken from the output of each machine.

M1	16.03	16.04	16.05	16.05	16.02	16.01	15.96	15.98	16.02	15.99
M2	16.02	15.97	15.96	16.01	15.99	16.03	16.04	16.02	16.01	16.00

- (a) Do you think the engineer is correct? Use  $\alpha = 0.05$ . Use  $\sigma_1^2 = \sigma_2^2$   
 (b) What is the probability value for this test?  
 (c) Do you think the engineer is correct? Use  $\alpha = 0.05$ .  $\sigma_1^2 \neq \sigma_2^2$
- 7.11 (cf. Montgomery, D. C., et al, 2001, 238). The deflection temperature under load for two different types of plastic pipe is being investigated. Two random sample of 15 pipe specimen are tested and the deflection temperatures observed are reported here (in  $^{\circ}\text{F}$ ).

Type 1	206	188	205	187	194	193	207	205
	185	189	213	192	210	194	178	192
Type 2	177	197	206	201	180	176	185	195
	200	197	192	198	188	189	203	192

- (a) Do the data support the claim that the deflection temperature under load for both types is same? Use  $\alpha = 0.05$   
 (b) Calculate the probability value for the test in part (a).
- 7.12 (cf. Montgomery, D. C., et al, 2001, 239). In semiconductor manufacturing, wet chemical etching is often used to remove silicon from the backs of wafers prior to metalization. The etch rate is an important characteristic in this process and known to follow a normal distribution. Two different etching solutions have been compared, using two random samples of 10 wafers from each solution. The observed etch rates are as follows (in mils/min):

Solution 1	9.9	9.4	9.3	9.6	10.2	10.6	10.3	10.0	10.3	10.1
Solution 2	10.2	10.6	10.7	10.4	10.5	10.0	10.2	10.7	10.4	10.3

Do the data support the claim that the mean etch rate is same for both the solutions?  
 Use  $\alpha = 0.05$

- (a) Assume  $\sigma_1^2 = \sigma_2^2$ .  
 (b) Assume  $\sigma_1^2 \neq \sigma_2^2$ .  
 (c) Calculate the  $p$ -value for the test in part (a).
- 7.13 Refer to Exercise 6.7, Conduct the most appropriate hypothesis test using a 0.05 significance level

- 7.14 Refer to Exercise 6.8, Conduct the most appropriate hypothesis test using a 0.01 significance level
- 7.15 Refer to Exercise 6.9, Conduct the most appropriate hypothesis test using a 0.01 significance level.
- 7.16 Refer to Exercise 6.10, Conduct the most appropriate hypothesis test using a .01 significance level.
- 7.17 Refer to Exercise 6.11, Conduct the most appropriate hypothesis test using a 1% significance level
- 7.18 Refer to Exercise 6.12, Conduct the most appropriate hypothesis test using a 0.05 significance level.
- 7.19 Refer to Exercise 6.13, Conduct the most appropriate hypothesis test using a .01 significance level.
- 7.20 Refer to Exercise 6.14, Conduct the most appropriate hypothesis test using a .01 significance level.
- 7.21 Refer to Exercise 6.15, Conduct the most appropriate hypothesis test using a 0.05 significance level.
- 7.22 Refer to Exercise 6.16, Conduct the most appropriate hypothesis test using a 0.05 significance level.
- 7.23 Refer to Exercise 6.17, Conduct the most appropriate hypothesis test using a 0.05 significance level.
- 7.24 Refer to Exercise 6.18, Conduct the most appropriate hypothesis test using a .05 significance level.
- 7.25 Refer to Exercise 6.22, Conduct the appropriate hypothesis test using a 0.05 significance level.
- 7.26 Refer to Exercise 6.23, Conduct the appropriate hypothesis test using a 0.10 significance level.
- 7.27 Refer to Exercise 6.24, Conduct the appropriate hypothesis test using a .05 significance level.
- 7.28 Refer to Exercise 6.25, Conduct the appropriate hypothesis test using a 0.01 significance level.
- 7.29 Refer to Exercise 6.26, Conduct the appropriate hypothesis test using a 0.05 significance level.
- 7.30 Refer to Exercise 6.27, Conduct the appropriate hypothesis test using a 0.10 significance level