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Q.1 The compressive strength of concrete has normal distribution with $\mu=2500$ psi and $\sigma=50$ psi. A random sample of $n=5$ specimens is selected, then:

- 6 (a) What is the sampling distribution for the sample mean
 \bar{X} has a normal dist with mean $\mu_{\bar{X}} = \mu = 2500$, Variance = $\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} = \frac{50^2}{5} = 500$
 or $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{50}{\sqrt{5}} = 22.3607$
 OR: $\bar{X} \sim N(\mu_{\bar{X}} = 2500, \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = 22.3607)$ (6 pts)

- 5 (b) Find the probability that the sample will have a sample mean diameter that falls in the interval from 2499 psi to 2510 psi.

$$\begin{aligned}
 &P(2499 < \bar{X} < 2510) = ? \\
 &= P\left(\frac{2499 - 2500}{22.3607} < \frac{\bar{X} - 2500}{22.3607} < \frac{2510 - 2500}{22.3607}\right) \quad \text{(2 pts)} \\
 &= P(-.04 < Z < 0.45) \\
 &= P(Z < 0.45) - P(Z < -.04) \quad \text{(2 pts)} \\
 &= .6736 - 0.4840 \\
 &= 0.1896 \quad \text{(1 pt)}
 \end{aligned}$$

- 9 Q2. The elasticity of a polymer is affected by the concentration of a reactant. When low concentration is used, the true mean elasticity is 55, and when high concentration is used the mean elasticity is 60. The standard deviation of elasticity is 4, regardless of concentration. If two random samples of size 36 are taken, find the probability that $P(\bar{X}_{high} - \bar{X}_{low} \geq 2.1)$

$$n_1 = n_2 = 36, \sigma_1 = \sigma_2 = 4$$

$$P(\bar{X}_{high} - \bar{X}_{low} > 2.1) = ?$$

$$\bar{X}_{high} - \bar{X}_{low} \approx N(\mu_{\bar{X}_{high} - \bar{X}_{low}} = \mu_{high} - \mu_{low} = 60 - 55 = 5, \sigma_{\bar{X}_{high} - \bar{X}_{low}} = \sqrt{\frac{4^2}{36} + \frac{4^2}{36}} = \sqrt{\frac{8}{9}} = 0.9428) \quad \text{(6 pts)}$$

$$\begin{aligned}
 P(\bar{X}_{high} - \bar{X}_{low} > 2.1) &= P\left(\frac{\bar{X}_{high} - \bar{X}_{low} - 5}{0.9428} > \frac{2.1 - 5}{0.9428}\right) \quad \text{(1 pt)} \\
 &= P(Z > -3.08) \\
 &= 1 - P(Z < -3.08) \\
 &= 1 - 0.0010 \quad \text{(1 pt)} \\
 &= 0.9990 \quad \text{(1 pt)}
 \end{aligned}$$