SOLUTIONS

King Fahd University of Petroleum & Minerals

Department of Mathematics & Statistics

STAT-319-Term073-Quiz3-B

Name: ID: Sec.: Serial:

Q.1 An optical inspection system is to distinguish among different part types. The probability of a correct classification of any part is 0.98. Suppose that **two** parts are inspected and that the classifications are independent. Let the random variable X denote the number of parts that are correctly classified.

a. Determine the probability distribution function of X.

Let: **C** a part is correctly classified, **N:** a part is NOT correctly classified

$S = \{CC, CN, NC, NN\}$ This implies that the values of X are: 0, 1, and 2. **(3-Points)**

$f(0) = P(NN) = (0.02)(0.02) = 0.0004$ **(1-Point)**

$f(1) = P(CN) + P(NC) = (0.98)(0.02) + (0.02)(0.98) = 0.0392$ **(1-Point)**

$f(2) = P(CC) = (0.98)(0.98) = 0.9604$ **(1-Point)**

The probability distribution of X is **(2-Points)**

X	**0**	**1**	**2**
$f(x)$	**0.0004**	**0.0392**	**0.9604**

b. If Y is a random variable given by $Y = X^2 + 1$ find the expected value of Y.

$E(Y) = E(X^2+1) = \sum_{all\ x} (x^2+1) f(x)$ **(1-Point)**

$= (0^2+1)(0.0004) + (1^2+1)(0.0392) + (2^2+1)(0.9604)$ **(1-Point)**

$= 0.0004 + 0.0784 + 4.802 = 4.8808$ **(1-Point)**

Q2. If the probability density function for a random variable is given by: $f(\theta) = \begin{cases} e^{-(\theta-3)} & , \theta > k \\ 0 & , elsewhere \end{cases}$

a. Find the value of K

$\int_k^\infty f(\theta) d\theta = 1$ **(1-Point)**

$\int_k^\infty e^{-(\theta-k)} d\theta = -e^{-(\theta-3)} \Big|_k^\infty = 1$

(2-Points)

$= -\left(0 - e^{-(k-3)}\right) = 1 \Rightarrow e^{-k+3} = 1$

$-k + 3 = 0 \Rightarrow k = 3$ **(1-Point)**

b. Find the distribution function $F(\theta)$

$F(\theta) = \int_3^\theta f(t) dt$ **(2-Points)**

$\int_3^\theta e^{-(t-3)} dt = -e^{-(t-3)} \Big|_3^\theta$ **(2-Points)**

$= -\left(e^{-(\theta-3)} - 1\right) = 1 - e^{-(\theta-3)}$ **(1-Point)**

So, $F(\theta) = \begin{cases} 1 - e^{-(\theta-3)}, & \theta \geq 3 \\ 0 & , \theta < 3 \end{cases}$