SOLUTIONS

King Fahd University of Petroleum & Minerals

Department of Mathematics & Statistics

STAT-319-Term073-Quiz3-A

Name: ID: Sec.: Serial:

Q.1 In a semiconductor manufacturing process, **two** wafers from a lot is tested. Each wafer is classified as pass or fail. Assume that the probability that a wafer passes the test is 0.8 and that wafers are independent.

a. Determine the probability distribution of X: the number of wafers from a lot that pass the test.

Let: **P** The wafer is classified as Pass, **F:** The wafer is classified as Fail

$S=\{PP, PF, FP, FF\}$ This implies that the values of X are: 0, 1, and 2. **(3-Points)**

$f(0)=P(FF)=(0.2)(0.2)=0.04$ **(1-Point)**

$f(1)=P(PF)+P(FP)=(0.8)(0.2)+(0.2)(0.8)=0.32$ **(1-Point)**

$f(2)=P(PP)=(0.8)(0.8)=0.64$ **(1-Point)**

The probability distribution of X is **(2-Points)**

X	**0**	**1**	**2**
$f(x)$	**0.04**	**0.32**	**0.64**

b. If Y is a random variable given by $Y=(X-1)^2$ find the expected value of Y.

$E(Y)=E\left((X-1)^2\right)=\sum_{all\ x}(x-1)^2 f(x)$ **(1-Point)**

$=(0-1)^2(0.04)+(1-1)^2(0.32)+(2-1)^2(0.64)$ **(1-Point)**

$=0.04+0+0.64=0.68$ **(1-Point)**

Q2. If the probability density function for a random variable X is: $f(x)=\begin{cases} e^{-(x-k)} & , x>4 \\ 0 & , elsewhere \end{cases}$

a. Find the value of K

$\int_4^{\infty} f(x)dx=1$ **(1-Point)**

$\int_4^{\infty} e^{-(x-k)}dx=-e^{-(x-k)}\Big|_4^{\infty}=1$ **(2-Points)**

$=-\left(0-e^{-(4-k)}\right)=1\Rightarrow e^{-4+k}=1$

$-4+k=0\Rightarrow k=4$ **(1-Point)**

b. Find the distribution function $F(x)$

$F(x)=\int_4^{x} f(t)dt$ **(2-Points)**

$\int_4^{x} e^{-(t-4)}dt=-e^{-(t-4)}\Big|_4^{x}$ **(2-Points)**

$=-\left(e^{-(x-4)}-1\right)=1-e^{-(x-4)}$ **(1-Point)**

So, $F(x)=\begin{cases} 1-e^{-(x-4)} & , x\geq 4 \\ 0 & , x<4 \end{cases}$