KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
DHAHRAN, SAUDI ARABIA

STAT 319: PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS

MidTerm Exam, Semester-073 (2008) - SOLUTIONS

Time: 4:00 pm to 6:00 pm. Saturday, August 2, 2008.

Tick (√) the box below corresponding to your Class **Section, Time, and Instructor**:

√	Section	Time	Instructor
	1	8.10-9.10am	Marwan Al-Momani
	2	9.20-10.20am	Mohammad H. Omar
	3	9.20-10.20am	Marwan Al-Momani
	4	10.30-11.30am	Mohammad H. Omar

Student Name: Key Solutions ID# Serial #

1) Mobiles are **NOT** allowed. Using mobile phones during exam is grounds for **cheating.**
2) Answer all questions.
3) You are allowed to use any scientific/electronic calculator.
4) Keep as many decimal places as possible while computing your answer (hint: use the memory button on your calculator). You should report at least 4 decimal places for your final answers (This will ensure your answer will have minimum rounding errors and will be close enough to the answer key).
5) On problem solving questions, **set-up the problem** and show **key important steps** to maximize your scores. For example,

   What is the total of the following data?

   12 14 6 8 10

   The following gives the necessary and sufficient steps to the solution.

   Total = (12+...+10) = 50.

   Notice that you can abbreviate (shorten) the steps to save time.

Question No	Marks	Marks Obtained	Comment
1	**20**		
2	**10**		
3	**8**		
4	**10**		
5	**13**		
6	**10**		
7	**12**		
8	**12**		
Total	**95**		

Note: You MUST show all key steps to obtain full credit for your answers.

STAT-319 : Marwan Al-Momani
* SOLUTIONS - MID-TERM - 073 *

Question One. (10+4+2+4=20-Points)

In a study of a galvanized coating process for large pipes, Standards call for an average coating weight of 200 lb per pipe. The following data are the coating weights for a random sample of 30 pipes:

193	196	198	200	202	202	202	203	204	204
204	204	204	205	206	206	206	207	208	208
208	212	212	212	213	215	216	216	218	218

Some summary information for this data is already calculated as follows:

$$\sum_{i=1}^{n} x_i = 6202 \quad \text{and} \quad \sum_{i=1}^{n} x_i^2 = 1283330$$

10

a. Calculate the following summary measures for the coating weight data:

i) Mode $= 204$ ① pt

ii) Median $n = 30 \Rightarrow \text{even} \Rightarrow \tilde{x} = \frac{x_{(15)} + x_{(16)}}{2} = \frac{206+206}{2} = 206$ } ② pts

iii) Mean $\bar{x} = \frac{\sum x_i}{n} = \frac{6202}{30} = 206.7333$ } ② pts

iv) The variance : $s^2 = \frac{\sum x_i^2 - n\bar{x}^2}{n-1} = \frac{1283330 - (30)(206.73333)^2}{30-1}$ } ③ pts

$= \frac{1170.28014}{29}$

$= 40.3545$ } ① pt

v) The standard deviation : $S = \sqrt{40.3545} = 6.3525$ } ① pt

4

b. Construct a stem-and-leaf plot of the coating weight data and based on this plot, **describe** the <u>shape</u> of the distribution.

Stem	leaves
19·	3
19*	6 9
20·	0 2 2 2 3 4 4 4 4 4
20*	5 6 6 6 7 8 8 8
21·	2 2 2 3
21*	5 6 6 8 8

① pt ① pt

shape : skewed left
② pts

2

c. Calculate the z-score for a coating weight of **199.**

$z = \frac{x - \bar{x}}{S} \Rightarrow z_{199} = \frac{199 - 206.7333}{6.3525} = -1.2174$ } ① pt

① pt ≈ -1.22

4

d. Does the coating weight distribution satisfy the **empirical rule**? (**Hint:** calculate the percentage within standard deviations from the mean and compare this with the rule.)

I. $[\bar{x} - S, \bar{x} + S] = [200.3808 , 213.0858]$ ② pts

There are 21 observations in the interval ⇒ Their percentage $= \frac{21}{30} \times 100\% = 70\%$ ① pt

∴ The data set does not satisfy the empirical rule. } ① pt

Question Two. (2+2+3+3=10-Points).

An order for a computer system can specify memory of 4, 8, or 12 gigabytes, and disk storage of 200, 300, or 400 gigabytes.

2

a. Describe the set of possible orders.

$S = \{(4,200), (4,300), (4,400), (8,200), (8,300), (8,400), (12,200), (12,300), (12,400)\}$

2 pts

2

b. Write the elements of the following events:
M: The computer system has **8** gigabytes **memory**
D: The computer system has **300** gigabytes **disk storage**.

$M = \{(8,200), (8,300), (8,400)\}$ 1 pt

$D = \{(4,300), (8,300), (12,300)\}$ 1 pt

3

c. Are the events M and D in part b **independent**? Please provide your **justification**.

$p(M) = \frac{3}{9} = \frac{1}{3}$, $p(D) = \frac{3}{9} = \frac{1}{3}$

$p(M \cap D) = p\{(8,300)\} = \frac{1}{9} = p(D) * p(M) = \frac{1}{3} * \frac{1}{3} = \frac{1}{9}$ 1 pt

1 pt

So, M & D are indep. 1 pt

3

d. Let the events F and G be defined as follows: (F) a computer system has 4 gigabyte memory and (G) a computer system has 400 gigabyte disk storage. Find the probability that an order for a computer is specified to have neither 4 gigabyte memory nor 400 gigabyte disk storage.

$p(F' \cap G') = p(F \cup G)' = 1 - p(F \cup G)$ 1 pt

$p(F \cup G) = p(F) + p(G) - p(F \cap G) = \frac{3}{9} + \frac{3}{9} - \frac{1}{9} = \frac{5}{9}$ } 1 pt

$\therefore p(F' \cap G') = 1 - \frac{5}{9} = \frac{4}{9}$ } 1 pt

Question Three. (4+4=8-Points).

The alignment between the magnetic tape and head in a magnetic tape storage system affects the performance of the system. Suppose that 10% of the read operations are degraded by skewed alignments, 5% of the read operations are degraded by off-center alignments, and the remaining read operations are properly aligned. The probability of a read error is 0.01 from a skewed alignment, 0.02 from an off-center alignment, and 0.001 from a proper alignment.

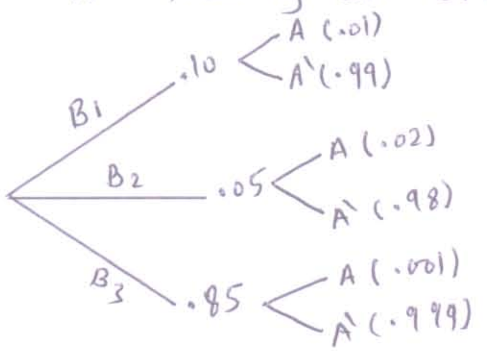
a. What is the probability of a read error?

Let B_1 : The read operations are degraded by skewed alignments → $p(B_1) = .10$

B_2 : 〃 〃 〃 〃 〃 〃 off-Center 〃 → $p(B_2) = .05$

B_3 : 〃 〃 〃 〃 properly aligned → $p(B_3) = 0.85$

A : Reading an error.



$$p(A) = p(A|B_1)\cdot p(B_1) + p(A|B_2)\cdot p(B_2) + p(A|B_3)\cdot p(B_3)$$
$$= (.01)(.10) + (.02)(.05) + (.001)(.85) \quad \text{③ pts}$$
$$= 0.001 + 0.001 + 0.00085$$
$$= 0.00285 \quad \text{① pt}$$

b. If a read error occurs, what is the probability that it is due to a skewed alignment?

$$p(B_1|A) = \frac{p(B_1 \cap A)}{p(A)} = \frac{p(A|B_1)\cdot p(B_1)}{p(A)} \quad \text{① pt}$$

$$= \frac{(.01)(.10)}{0.00285} = \frac{0.001}{0.00285} \quad \text{② pts}$$

$$= 0.3509. \quad \text{① pt}$$

Question Four. (3+2+3+2=10-Points).

A disk-drive manufacturer estimates that in five years a storage device with 1 terabyte of capacity will sell with probability of 0.5, a storage device with 500 gigabytes capacity will sell with a probability of 0.3, and a storage device with 100 gigabytes capacity will sell with probability of 0.2. The revenue associated with the sales in these years is estimated to be \$50 million, \$26 million, and \$10 million, respectively. Let X be the revenue of storage devices during these years.

3

a. Determine the **probability distribution** of X.

The possible values of X (in millions) are : 50, 26, 10 } 1 pt

$$\left.\begin{aligned} f(50) &= p(X=50) = 0.5 \\ f(26) &= p(X=26) = 0.3 \\ f(10) &= p(X=10) = 0.2 \end{aligned}\right\} \text{1 pt}$$

The prob. dist. of X is:

x	10	26	50
f(x)	.2	.3	.5

1 pt

2

b. Determine the **expected** value of X.

$$E(X) = \sum_{\text{all } x} x\, f(x) = (10)(.2) + (26)(.3) + (50)(.5) \ \} \text{ 1 pt}$$

$$= 2 + 7.8 + 25$$

$$= 34.8 \text{ millions.} \ \} \text{ 1 pt}$$

3

c. If $g(X)$ is a function of random variable X, where $g(X) = \dfrac{\sqrt{X-1}}{2} + 3$, find $\mu_{g(X)}$.

$$\left.\begin{aligned} \mu_{g(X)} = E(g(X)) &= \sum_{\text{all } x} g(x)\, f(x) \\ &= \sum_{\text{all } x} \left(\frac{\sqrt{x-1}}{2} + 3\right) f(x) \end{aligned}\right\} \text{1 pt}$$

$$\left.\begin{aligned} &= \left(\frac{\sqrt{10-1}}{2}+3\right)(.2) + \left(\frac{\sqrt{26-1}}{2}+3\right)(.3) + \left(\frac{\sqrt{50-1}}{2}+3\right)(.5) \\ &= 0.9 + 1.65 + 3.25 \end{aligned}\right\} \text{1 pt}$$

$$= 5.8 \ \} \text{ 1 pt}$$

2

d. Find the **cumulative distribution function** of X.

$$F(x) = P(X \le x) = \begin{cases} 0 & , \text{if } x < 10 \\ 0.2 & , \text{if } 10 \le x < 26 \\ 0.5 & , \text{if } 26 \le x < 50 \\ 1 & , \text{if } x \ge 50 \end{cases}$$

2 pts

Question Five. (3+4+6 =13-Points).

The probability density function of a random variable X is given by:

$$f(x)=\begin{cases} kx^2, & -1<x<1 \\ 0 & ,elsewhere \end{cases}$$

3

a. Find the value of k

$\int_{-1}^{1} f(x)\,dx = 1 \Rightarrow \int_{-1}^{1} kx^2\,dx = 1 \}$ 1 pt

$\frac{kx^3}{3}\Big|_{-1}^{1} = 1 \Rightarrow \frac{k}{3}(1+1) = 1 \}$ 1 pt

$\frac{2k}{3} = 1 \Rightarrow k = \frac{3}{2} \}$ 1 pt

4

b. If $h(X)=e^{X^3}$, find $E(h(X))$.

$E(h(X)) = \int_{-1}^{1} h(x)\,f(x)\,dx$ 1 pt

$= \int_{-1}^{1} e^{x^3}\cdot\frac{3}{2}x^2\,dx$

Let $u = x^3 \Rightarrow du = 3x^2\,dx$

$\Rightarrow dx = \frac{du}{3x^2}$

when $x=-1 \Rightarrow u=-1$

$x=1 \Rightarrow u=1$

$= \int_{-1}^{1} e^{u}\cdot\frac{3x^2}{2}\cdot\frac{du}{3x^2}$

$= \frac{1}{2}\int_{-1}^{1} e^{u}\,du$

$= \frac{1}{2}e^{u}\Big]_{-1}^{1}$

$= \frac{1}{2}(e - e^{-1})$

2 pts

$= 1.1752 \}$ 1 pt

6

c. Determine the **variance** of the random variable X

$Var(X) = \sigma^2 = E(X^2) - (E(X))^2 \}$ 1 pt

$E(X) = \int_{-1}^{1}\frac{3}{2}x^2\cdot x\,dx = \int_{-1}^{1}\frac{3}{2}x^3\,dx = 0$ (odd func. and symmetric interval) 2 pts

$E(X^2) = \int_{-1}^{1} x^2\cdot\frac{3}{2}x^2\,dx = \frac{3}{2}\cdot\int_{-1}^{1} x^4\,dx \}$ 1 pt

$= \frac{3}{2}\,\frac{x^5}{5}\Big]_{-1}^{1} = \frac{3}{10}(1+1) = \frac{6}{10} = .6 \}$ 1 pt

$\therefore\ \sigma^2 = (.6) - (0)^2 = 0.6$ 1 pt

Question Six. (2+4+4=10-Points).

A new automated production process for automotive airbags averages 1.5 breakdowns per week. Because of the cost associated with the breakdown, the possibility of having two or more breakdowns in two weeks is very undesirable.

$\boxed{2}$ a. Compute the **average number** of breakdowns in **two** weeks.

$\lambda = 1.5/\text{week}$, $t = 2$ (two weeks) . Poisson dist.
1 pt

$$\text{Average} = \mu = E(X) = \lambda t = (1.5)(2) = 3 \quad \text{1 pt}$$

$\boxed{4}$ b. What is the probability of having **two or more breakdowns** in **two** weeks?

$\lambda = 1.5$, $t = 2 \Rightarrow \lambda t = 3$

$$p(X \geq 2) = 1 - p(X < 2) \quad \text{1 pt}$$
$$= 1 - p(X \leq 1)$$
$$= 1 - (p(X=0) + p(X=1))$$
$$= 1 - \left(\frac{3^0 * e^{-3}}{0!} + \frac{3^1 * e^{-3}}{1!} \right) \quad \text{1 pt}$$
$$= 1 - e^{-3}(1+3) = 1 - 4e^{-3} \quad \text{1 pt}$$
$$= 1 - 0.1991$$
$$= 0.8009 \quad \text{1 pt}$$

$\boxed{4}$ c. Compute the probability of **no breakdowns** in **three** weeks.

$\lambda = 1.5$, $t = 3 \Rightarrow \lambda t = (1.5)(3) = 4.5$ 1 pt

$$p(X=0) = \frac{(4.5)^0 e^{-4.5}}{0!} \quad \text{1 pt}$$
$$= e^{-4.5} \quad \text{1 pt}$$
$$= 0.0111 \quad \text{1 pt}$$

Question Seven. (5+4+3=12-Points).

A machine fills containers with a particular product. The standard deviation of filling weights is known from past data to be 0.6 ounce. Assume the filling weights have a normal distribution.

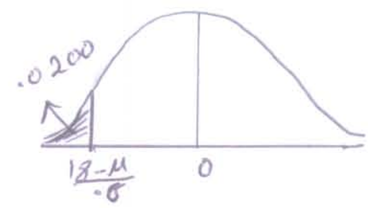
5

a. If only 2% of the containers hold less than 18 ounces of the filling weights, what is the **mean filling weight** of the machine? (Hint: what must μ equal?)

$\sigma = 0.6 \quad , \mu = ?$

$p(X < 18) = 0.02$ ① pt

$p\left(\frac{X-\mu}{.6} < \frac{18-\mu}{.6}\right) = 0.02$ } ① pt



$\Rightarrow \frac{18-\mu}{.6} = -2.05$ } ① pt

$18 - \mu = (.6)(-2.05)$
$\Rightarrow \mu = 18 + 1.23$ } ① pt

$\mu = 19.23$ } ① pt

$\therefore \quad X \sim n(\mu = 19.23, \sigma = 0.6)$

4

b. Assuming the mean filling weight is 20 ounces; find the probability of finding a container that holds **more than 21.2** ounces of filling weights.

$X \sim n(\mu = 20, \sigma = 0.6)$

$p(X > 21.2) = ?$ ① pt

$= p\left(\frac{X-20}{.6} > \frac{21.2-20}{.6}\right)$ } ① pt

$= p(Z > 2)$
$= 1 - p(Z < 2)$ } ① pt

$= 1 - 0.9772$
$= 0.0228$ } ① pt

3

c. Assuming the mean filling weight is 20 ounces and 21.2 is considered the maximum capacity for filling weights, what is the probability of finding that the **fourth container will hold more than** the maximum capacity of filling weights?

① pt $X \sim \text{Geometric}(p = 0.0228) \Rightarrow q = 1 - .0228 = .9772$

$p(X = 4) = (.0228)(.9772)^3$ } ① pt

$= 0.0213$ } ① pt

Question Eight. (3+4+5=12-Points).

The lifetime (in hours) of an electronic smartcard reader device is a random variable with the following probability density function.

$$f(x) = \frac{1}{50}e^{-x/50} \qquad \text{for } x \geq 0$$

$\boxed{3}$

a. What is the **expected lifetime** of the smartcard reader device?

$X \sim Exp.(\beta = 50) \Rightarrow \mu = \beta = 50$ } 3 pts

OR.

$\mu = E(X) = \int_0^\infty x \cdot f(x)\,dx = \frac{1}{50}\int_0^\infty x\, e^{-X/50}\,dx$

$u = x \qquad dv = e^{-X/50}$

$du = dx \qquad v = -50\, e^{-X/50}$

$\mu = \frac{1}{50}[-50x\, e^{-X/50} \Big|_0^\infty + 50\int_0^\infty e^{-X/50}\,dx]$

$= \frac{1}{50}[50 \cdot (-e^{-X/50} \cdot 50) \Big]_0^\infty] = (0 + 50 e^0) = 50$ } 1 pt

} 2 pts

OR

$\boxed{4}$

b. What is the probability that the smartcard reader device will operate **80.47 or more hours** before failure?

$p(X > 80.47) = \int_{80.47}^\infty \frac{1}{50} e^{-X/50}\,dx$ } 1 pt

$= -e^{-X/50} \Big]_{80.47}^\infty$ } 1 pt

$= -(0 - e^{-1.6094})$ } 1 pt

$= 0.2000$ } 1 pt

$\boxed{5}$

c. If five of these smartcard reader devices are used in a university building, what is the probability that **at least 4** of them will operate **80.47 or more hours** before failing?

1 pt $X \sim binomial(n = 5, p = .2) \Rightarrow q = 1 - .2 = .8$

$p(\text{at least } 4) = p(X \geq 4)$ 1 pt

$= p(X=4) + p(X=5)$

$= \binom{5}{4}(.2)^4(.8)^1 + \binom{5}{5}(.2)^5(.8)^0$ } 2 pts

$= 0.0064 + 0.00032$

$= 0.0067$ } 1 pt