

Sampling distribution

$$\mu_{\bar{X}} = \mu, \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

$$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2,$$

$$\sigma_{\bar{X}_1 - \bar{X}_2}^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

Student t-distribution

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}, \quad df = n - 1,$$

Chi-Square Distribution:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}, \quad df = n - 1$$

F-distribution

$$F = \frac{s_1^2}{s_2^2}, \quad df_1 = n_1 - 1, df_2 = n_2 - 1$$

$$F_{\alpha, n_1, n_2} = \frac{1}{F_{1-\alpha, n_2, n_1}}$$

Confidence Interval Estimation

1) single Sample: C.I for μ

a. when σ known: $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

b. when n small: $\bar{x} \pm t_{\alpha/2} \frac{S}{\sqrt{n}}$

c. Large sample: $\bar{x} \pm z_{\alpha/2} \frac{S}{\sqrt{n}}$

d. Error = $e = \frac{z_{\alpha/2} \sigma}{\sqrt{n}}$, $e = \frac{t_{\alpha/2} S}{\sqrt{n}}$ or

$$e = \frac{z_{\alpha/2} S}{\sqrt{n}}$$

e. Required sample size = $n = \left(\frac{z_{\alpha/2} \sigma}{e} \right)^2$

2) single Sample : C.I for P

a. $\hat{P} \pm z_{\alpha/2} \sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$

b. Error = $z_{\alpha/2} \sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$

c. Required sample size = $n = \frac{Z_{\alpha/2}^2 \hat{P}(1-\hat{P})}{e^2}$

3) One sample: C.I for σ

$$\frac{(n-1)S^2}{\chi_{\alpha/2}^2} < \sigma < \frac{(n-1)S^2}{\chi_{1-\alpha/2}^2}$$

4) Two Samples :C.I for $\mu_1 - \mu_2$

1) If σ_1 and σ_2 are known

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

2) Large samples

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

3) Small samples:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \text{ where}$$

$$S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$df = n_1 + n_2 - 2$$

4) Paired samples: C.I for μ_D

$$\bar{d} \pm t_{\alpha/2} \frac{S_d}{\sqrt{n}}, \quad df = n - 1$$

5) Two Samples :C.I for $P_1 - P_2$

$$(\hat{P}_1 - \hat{P}_2) \pm z_{\alpha/2} \sqrt{\frac{\hat{P}_1 \hat{q}_1}{n_1} + \frac{\hat{P}_2 \hat{q}_2}{n_2}}, \text{ where}$$

$$\hat{P}_i = \frac{X_i}{n_i}, \quad \hat{q}_i = 1 - \hat{P}_i$$

Testing Hypotheses

1) single Sample: Testing about μ

a. when σ known: $Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

b. when n small: $t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}, df = n - 1$

c. Large sample: $Z = \frac{\bar{x} - \mu_0}{S/\sqrt{n}}$

2) single Sample : testing about P

Large sample test:

$$Z = \frac{X - np_0}{\sqrt{np_0q_0}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0q_0}{n}}}$$

3) Two Samples :Testing about $\mu_1 - \mu_2$ **a. If σ_1 and σ_2 are known**

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

b. Large Samples:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

c. Small samples:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

d. Paired samples: Testing about μ_D

$$t = \frac{\bar{d} - d_0}{s_d / \sqrt{n}}$$

4) Two Samples :Testing about : $P_1 - P_2$ **Large Samples under $H_0 : P_1 - P_2 = 0$**

$$Z = \frac{\hat{P}_1 - \hat{P}_2 - 0}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \text{ where}$$

$$\hat{p} = \frac{n_1 \hat{P}_1 + n_2 \hat{P}_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$$

Simple linear Regression**1) Estimated regression model:**

$$\hat{y} = a + bx, \text{ where:}$$

$$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

$$= \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

$$\text{And } a = \bar{y} - b \bar{x}$$

2) Sum of squares:

$$\text{a. } S_{XX} = \sum(x - \bar{x})^2 = \sum x^2 - (\sum x)^2 / n$$

$$\text{b. } S_{YY} = \sum(y - \bar{y})^2 = \sum y^2 - (\sum y)^2 / n$$

$$S_{XY} = \sum(x - \bar{x})(y - \bar{y})$$

$$\text{c. } = \sum xy - (\sum x)(\sum y) / n$$

$$\text{d. } SSE = \sum(y_i - \hat{y}_i)^2 = S_{YY} - b S_{XY}$$

$$\text{e. } SST = S_{YY} = \sum(y_i - \bar{y})^2$$

$$\text{3) } S^2 = MSE = \frac{SSE}{n-2}$$

4) Inference about the regression coefficients

$$\text{a. C.I for } \beta : b \pm t_{\alpha/2} \frac{S}{\sqrt{S_{XX}}}, df = n - 2$$

$$\text{b. C.I for } \alpha : a \pm \frac{t_{\alpha/2} S \sqrt{\sum x^2}}{\sqrt{n S_{XX}}}$$

$$\text{c. Testing about } \beta : t = \frac{b - \beta_0}{S / \sqrt{S_{XX}}}$$

$$\text{d. Testing about } \alpha : t = \frac{a - \alpha_0}{S \sqrt{\frac{\sum x^2}{n S_{XX}}}}$$

5) C. I for the mean response $\mu_{Y|x_0}$

$$\hat{y}_0 \pm t_{\alpha/2} S \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{XX}}}$$

6) P.I for a single response y_0 is:

$$\hat{y}_0 \pm t_{\alpha/2} S \sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{XX}}}$$

7) Coefficient of determination

$$R^2 = 1 - \frac{SSE}{SST}$$

8) Correlation coefficient

$$r = b \frac{\sqrt{S_{XX}}}{\sqrt{S_{YY}}} = \frac{S_{XY}}{\sqrt{S_{XX} S_{YY}}}, r^2 = R^2$$

9) Testing about the population correlation coefficient:

$$t = \frac{b}{S / \sqrt{S_{XX}}} = \frac{r \sqrt{n-2}}{\sqrt{1-r^2}}, df = n - 2$$