✻SOLUTIONS✻

King Fahd University of Petroleum & Minerals
Department of Mathematical Science
STAT319-Term041
Quiz #3   Section: 5
Name: ID: Serial:

A materials researcher wishes to use the following sample curing-time data to compare two methods for applying an adhesive;

Method A	14	16	13	15	16	13	14	14	15	11
Method B	8	9	12	6	7	7	10	5	14	10

Given that:

$$\bar{X}_A = 14.1 \;,\; \bar{X}_B = 8.8 \;,\; \bar{d} = 5.3 \;,\; s_A = 1.5239 \;,\; s_B = 2.7809 \;,\; s_d = 3.3682$$

a) How large a sample is required if we want to be 98% confident that our estimate of μ_A is off by less than 0.95?

$$1-\alpha = 0.98 \;\Rightarrow\; \alpha = .02 \;\Rightarrow\; Z_{\alpha/2} = Z_{.01} = 2.33 \;,\; e = 0.95$$

$$n = \frac{Z_{\alpha/2}^2 \, \sigma^2}{e^2} = \frac{Z_{\alpha/2}^2 \cdot S^2}{e^2} = \frac{(2.33)^2 (1.5239)^2}{(.95)^2} = 13.969$$

$$n \approx 14$$

②

b) Construct a 98% CI for the difference between the two method means. Assume normal populations with equal variances.

$$\sigma_1^2 = \sigma_2^2 = \sigma^2 \;(\text{unknown})$$

$$1-\alpha = .98 \;\Rightarrow\; \alpha = .02 \;\Rightarrow\; t_{\alpha/2, \, n_1+n_2-2} = t_{.01, \, 18} = 2.552$$

$$Sp \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = \sqrt{\frac{(10-1)(1.5239)^2 + (10-1)(2.7809)^2}{10+10-2}} \sqrt{\frac{1}{10} + \frac{1}{10}} = 1.0028$$

④

A 98% C.I. for $\mu_A - \mu_B$ is:

$$14.1 - 8.8 \pm (2.552)(1.0028)$$

$$2.741 \leq \mu_A - \mu_B \leq 7.859$$

c) Construct a 98% CI for the variance of method B. State necessary assumptions.

$$1-\alpha = 0.98 \;\Rightarrow\; \alpha = .02$$

$$\chi^2_{\alpha/2, \, n-1} = \chi^2_{.01, 9} = 21.666 \;,\; \chi^2_{1-\alpha/2, \, n-1} = \chi^2_{.99, 9} = 2.088$$

③

A 98% C.I. for σ^2 is:

$$\frac{(10-1)(2.7809)^2}{21.666} < \sigma^2 < \frac{(10-1)(2.7809)^2}{2.088}$$

$$3.212 < \sigma^2 < 33.334$$

Assumption: The sample is from normal population   ①