SOLUTIONS

King Fahd University of Petroleum & Minerals

Department of Mathematics & Statistics - STAT-319-Term071-**Quiz5**

Name: ID: Sec.: Serial:

Q1. The amount of time that a customer spends waiting at an airport check-in counter is a random variable with mean $\mu = 8.2$ minutes and standard deviation $\sigma = 1.5$ minutes. Suppose that a random sample of $n = 49$ customers is observed.

a. What is the sampling distribution of the sample mean?

$\overline{X}$ has approximately normal distribution with

1. Mean = $\mu_{\overline{X}} = \mu = 8.2$

2. Standard Error = $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{1.5}{7} = 0.2143$

Or: $\overline{X} \approx N\left(\mu_{\overline{X}} = \mu = 8.2, \sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{1.5}{7} = 0.2143\right)$

b. Find the probability that the average time waiting in line for these customers is less than 10 minutes

$$P\left(\overline{X} < 10\right) \simeq P\left(\frac{\overline{X} - 8.2}{0.2143} < \frac{10 - 8.2}{0.2143}\right) = P\left(Z < 8.40\right) \approx 1$$

Q2. A random sample of size $n_1 = 16$ is selected from a normal population with a mean of $\mu_1 = 75$ and a standard deviation of $\sigma_1 = 8$ 8. A second random sample of size $n_2 = 9$ is taken from another normal population with mean $\mu_2 = 70$ and standard deviation $\sigma_2 = 12$. Let $\overline{X}_1$ and $\overline{X}_2$ be the two sample means. Find

a. The probability that $\overline{X}_1 - \overline{X}_2$ exceeds 4

$$\overline{X}_1 - \overline{X}_2 \sim N\left(\mu_{\overline{X}_1 - \overline{X}_2} = 75 - 70 = 5, \ \sigma_{\overline{X}_1 - \overline{X}_2} = \sqrt{\frac{(8)^2}{16} + \frac{(12)^2}{9}} = \sqrt{20} = 4.4721\right)$$

$$P\left(\overline{X}_1 - \overline{X}_2 > 4\right) = p\left(\frac{\overline{X}_1 - \overline{X}_2 - 5}{\sqrt{20}} > \frac{4 - 5}{\sqrt{20}}\right) = P\left(Z > -0.22\right) = 0.5871$$

b. The probability that $3.5 \le \overline{X}_1 - \overline{X}_2 \le 5.5$

$$P\left(3.5 \le \overline{X}_1 - \overline{X}_2 \le 5.5\right) = P\left(\frac{3.5 - 5}{\sqrt{20}} \le \frac{\overline{X}_1 - \overline{X}_2 - 5}{\sqrt{20}} \le \frac{5.5 - 5}{\sqrt{20}}\right)$$

$$= P\left(-0.34 < Z < 0.11\right)$$

$$= 0.5438 - 0.3669 = 0.1769$$