KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
DHAHRAN, SAUDI ARABIA

STAT 319: PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS

Second Major Exam, Semester 063 SOLUTIONS
Saturday August 4, 2007

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Duration: 90 Minutes

Surname: ID# Serial:

Question No	Full Marks	Marks Obtained
Q1	**10**	
Q2	**6**	
Q3	**8**	
Q4	**8**	
Q5	**12**	
Q6	**9**	
Q7	**12**	
Total	**65**	

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Question1. (2+2+6=10-Points)

A particularly long traffic light on your morning commute is green 20% of the time that you approach it. Assume that each morning represents an independent trial.

a. Over five mornings, what is the probability that the light is green on exactly one day?

$p = 0.2$, $n = 5$, $X = 1$, $q = 1 - .2 = .8$

$$P(X=1) = \binom{5}{1}(.2)^1(.8)^4 = 0.4096 \quad \Big\} \text{ 2 pts}$$

b. Over 20 mornings, what is the expected number of days that the light is **not** green?

$n = 20$, $p = 1 - 0.2 = 0.8$ 1 pt

$E(X) = np = (20)(.8) = 16$ days 1 pt

c. Suppose that a sample of 100 days is selected at random, what is the probability that the light is green in more than 18 but less than 25 days?

$X \approx n(\mu = np, \sigma = \sqrt{npq})$

$\mu = np = (100)(.2) = 20$

$\sigma = \sqrt{npq} = \sqrt{(100)(.2)(.8)} = \sqrt{16} = 4 \therefore X \approx n(\mu = 20, \sigma = 4) \Big\}$ 1 pt

$$\begin{aligned}
p(18 < X < 25) &= p(19 \le X \le 24) \quad \text{1 pt}\\
&= p(18.5 \le X \le 24.5) \quad \text{1 pt}\\
&\simeq p\left(\frac{18.5-20}{4} \le \frac{X-20}{4} \le \frac{24.5-20}{4}\right) \quad \text{1 pt}\\
&\simeq p(-0.38 \le Z \le 1.13)\\
&= p(Z \le 1.13) - p(Z \le -.38)\\
&= 0.8708 - 0.3520 \quad \Big\} \text{ 1 pt}\\
&= 0.5188. \quad \Big\} \text{ 1 pt}
\end{aligned}$$

Question2.(2+4=6-Points)

A quality-control inspector accepts shipments whenever a sample of size 5 contains no defectives, and he rejects otherwise.

a. What is the probability that he will <u>accept</u> a poor shipment of 20 items in which 4 are defective?

$$p(\text{Accept}) = p(X=0) = \frac{\binom{4}{0}\binom{16}{5}}{\binom{20}{5}} = \frac{(1)(4368)}{15504} \text{ 1 pt}$$

$$= 0.2817 \quad \Big\} \text{ 1 pt}$$

5 | 16 N | 4 D

N: Not deffective
D: Defective

b. What is the probability that he will <u>reject</u> a good shipment of 40 items in which 2 are defective?

$p(\text{Reject}) = p(X \ge 1)$ 1 pt

5 | 38 N | 2 D

$$\begin{aligned}
&= 1 - p(X<1) = 1 - p(X=0) \quad \Big\} \text{ 1 pt}\\
&= 1 - \frac{\binom{2}{0}\binom{38}{5}}{\binom{40}{5}} \quad \Big\} \text{ 1 pt}\\
&= 1 - 0.7678 = 0.2372 \quad \Big\} \text{ 1 pt}
\end{aligned}$$

Question 3(4+4=8-Points)

The probability of a successful optical alignment in the assembly of an optical data storage product is 0.8. Assume the trials are independent.

a. What is the probability that at most one successful alignment among 4 trials?

$n = 4$, $p = 0.8$, binomial $(n = 4, p = .8)$

$p(X \le 1) = p(X=0) + p(X=1)$ } ① pt

$= \binom{4}{0}(.8)^0(.2)^4 + \binom{4}{1}(.8)^1(.2)^3$ } ② pts

$= 0.0016 + 0.0256 = 0.0272$ } ① pt

b. What is the probability that the first successful alignment requires at most four trials?

Geometric with $p = .8$, $q = 1 - .8 = .2$

$p(X \le 4) = p(X=1) + p(X=2) + p(X=3) + p(X=4)$ ① pt

$= (.8)(.2)^0 + (.8)(.2)^1 + (.8)(.2)^2 + (.8)(.2)^3$ } ② pts

$= (.8)[1 + .2 + 0.04 + 0.008]$

$= 0.9984.$ } ① pt.

Question 4(4+4=8-Points)

Calls to a toll-free telephone hotline service are made randomly and independently at an expected rate of 2 per minute.

a. What is the probability that the hotline receives more than 2 calls in the next three minutes?

Poisson with $\lambda = 2$, $t = 3 \Rightarrow \lambda t = 6$ } ① pt

$p(X > 2) = 1 - p(X \le 2)$

$= 1 - [p(X=0) + p(X=1) + p(X=2)]$

$= 1 - \left[\frac{6^0 e^{-6}}{0!} + \frac{6^1 e^{-6}}{1!} + \frac{6^2 e^{-6}}{2!}\right]$ } ② pts

$= 1 - [e^{-6} + 6e^{-6} + 18e^{-6}] = 1 - 0.0620 = 0.9380$

① pt

b. In sample of size n = 49 minutes what is the probability that the sample mean will be less than 1.5.

$n = 49$ (large sample size) $\lambda t = 2$

$\bar{X} \approx n\left(\mu_{\bar{X}} = \mu = \lambda t = 2,\ \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{2}}{7}\right) \longrightarrow$ ② pts

$p(\bar{X} < 1.5) = p\left(\frac{\bar{X} - 2}{\sqrt{2}/7} < \frac{1.5 - 2}{\sqrt{2}/7}\right)$ } ① pt

$= p(Z < -2.47)$

$= 0.0066$ } ① pt

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Question 5 (2+4+3+3=12-Points)

The life of a semiconductor laser at a constant power is normally distributed with a mean of 7000 hours and a standard deviation of 600 hours.

a. What is the probability that a laser fails before 5500 hours?

$X \sim N(\mu = 7000, \sigma = 600)$

$$\left.\begin{aligned} p(X < 5500) &= p\left(\frac{X-7000}{600} < \frac{5500-7000}{600}\right) \\ &= p(Z < -2.50) \end{aligned}\right\} \text{1 pt}$$

$$= 0.0062 \quad \} \text{ 1 pt}$$

b. What is the life in hours that 95% of the lasers exceed?

We need k such that: $p(X > k) = 0.95$ 1 pt

$p(X > k) = 0.95$

$\Rightarrow p(X < k) = 1 - .95 = 0.0500$

$p\left(\frac{X-7000}{600} < \frac{k-7000}{600}\right) = .0500 \Rightarrow p\left(Z < \frac{k-7000}{600}\right) = .05$ } 1 pt

1 pt

$\therefore \frac{k-7000}{600} = -1.645 \Rightarrow k = 7000 + 600(-1.645) = 6013$ hours

1 pt

c. If three lasers are used in a product and they are assumed to fail independently, what is the probability that all three are still operating after 7000 hours?

binomial: $n = 3$, p = prob. that of a semiconductor still operating after 7000

$p = p(X > 7000) = p(Z > 0) = .5 \Rightarrow q = 1 - p$

1 pt

$$\left.\begin{aligned} p(X = 3) &= \binom{3}{3}(.5)^3(.5)^0 \\ &= 0.125 \end{aligned}\right\} \text{2 pts}$$

d. Suppose that a sample of size n = 16 semiconductors was randomly selected, what is the probability that the sample mean will be less than 6800 hours

$\bar{X} \sim n\left(\mu_{\bar{X}} = \mu = 7000,\ \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{600}{4} = 150\right)$ 1 pt

$$\left.\begin{aligned} p(\bar{X} < 6800) &= p\left(\frac{\bar{X}-7000}{150} < \frac{6800-7000}{150}\right) \\ &= p(Z < -1.33) \end{aligned}\right\} \text{1 pt}$$

$$= 0.0918 \quad \} \text{ 1 pt}$$

Question 6(2+4+3=9-Points)

The life time of batteries manufactured by a factory has an exponential distribution with mean 320 hours. A battery is selected randomly from the product of the factory. Then:

a. Find the probability that the battery will work at most 300 hours.

$$\beta = 320 \text{ hrs} \Rightarrow f(x) = \begin{cases} \frac{1}{320} e^{-x/320} & , x>0 \\ 0 & , \text{o.w} \end{cases}$$

$$p(X \le 300) = \int_0^{300} \frac{1}{320} e^{-x/320}\, dx \quad \} \text{ 1 pt}$$

$$= -e^{-x/320} \Big|_0^{300} = -e^{-0.9375} + 1 = 0.6084 \quad \} \text{ 1 pt}$$

b. Find the probability that the battery will work more than 400 hours given that it has worked more than 360 hours.

Let A: The battery works more than 400 hrs

B: " " " " " 360 hrs

0 400

0 360

$$p(A|B) = \frac{P(A \cap B)}{P(B)} \qquad A \cap B = [400, \infty) \} \text{ 1 pt}$$

$$= \frac{p(X > 400)}{p(X>360)} = \frac{\int_{400}^{\infty} \frac{1}{320} e^{-x/320}\, dx}{\int_{360}^{\infty} \frac{1}{320} e^{-x/320}\, dx}$$

$$= \frac{-e^{-x/320} \Big|_{400}^{\infty}}{-e^{-x/320} \Big|_{360}^{\infty}} = \frac{0 + e^{-1.25}}{0 + e^{-1.125}} \quad \begin{matrix} \rightarrow \text{1 pt} \\ \rightarrow \text{1 pt} \end{matrix}$$

$$= \frac{0.2865}{0.3247} = 0.8824 \quad \} \text{ 1 pt}$$

c. Find the median of the life time of the battery.

Let m be the median, then

$$p(X < m) = 0.5 \quad \} \text{ 1 pt}$$

$$\Rightarrow \int_0^m \frac{1}{320} e^{-x/320}\, dx = 0.5$$

$$-e^{-x/320} \Big|_0^m = .5$$

$$-e^{-m/320} + 1 = .5 \Rightarrow e^{-m/320} = .5 \quad \} \text{ 1 pt}$$

$$\frac{-m}{320} = \ln(.5) = -0.6931$$

$$\therefore m = (-320)(-.6931) = 221.8071 \text{ hrs.} \quad \} \text{ 1 pt}$$

Question 7(3+5+4 =12-Points)

The distribution of tensile strength of the first grade of aluminum spars used in manufacturing the wing of a commercial transport aircraft has a mean 90 kilograms per square millimeter and a standard deviation of 6.2 kilogram per square millimeter, while the second grade of aluminum spars has a mean of 75 kilograms per square millimeter and a standard deviation of 8.3 kilograms per square millimeter. If two random samples were selected from the first and second grades of sizes $n_1 = 40$ and $n_2 = 50$ respectively. Assume the two grades are independent.

a. Find the mean and the variance and the sampling distribution of $\overline{X_1} - \overline{X_2}$

First grade: $\mu_1 = 90$, $\sigma_1 = 6.2$, $n_1 = 40$

Second grade: $\mu_2 = 75$, $\sigma_2 = 8.3$, $n_2 = 50$

I. $\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2 = 90 - 75 = 15$ } 1 pt

II. $\sigma^2_{\bar{X}_1 - \bar{X}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{(6.2)^2}{40} + \frac{(8.3)^2}{50}$

$= 0.961 + 1.3778 = 2.3388$ 1 pt

III. $\bar{X}_1 - \bar{X}_2 \sim n(\mu_{\bar{X}_1 - \bar{X}_2} = 15,\ \sigma_{\bar{X}_1 - \bar{X}_2} = 1.5293)$ } 1 pt

b. What is the probability that the first sample mean exceeds the second sample mean by at least 12.5 but less than 16 kilograms per square millimeter?

$p(12.5 \le \bar{X}_1 - \bar{X}_2 \le 16)$

$= p\left(\frac{12.5 - 15}{1.5293} \le \frac{(\bar{X}_1 - \bar{X}_2) - 15}{1.5293} \le \frac{16 - 15}{1.5293}\right)$

$= p(-1.63 \le Z \le 0.65)$ } 2 pts

$= p(Z \le .65) - p(Z < -1.63)$

$= 0.7422 - 0.0516$ } 2 pts

$= 0.6906$ } 1 pt

c. Assume that the distribution of tensile strength of the first grade of aluminum spars is normal; find the probability that the sample variance for a sample of size 30 will exceed 61.8923.

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$ 1 pt

$p(S^2 > 61.8923) = p\left(\frac{(29)S^2}{(6.2)^2} > \frac{(29)(61.8923)}{(6.2)^2}\right)$ } 2 pts

$= p(\chi^2_{29} > 46.693) \simeq 0.02$ } 1 pt