**KING FAHD UNIVERSITY OF PETROLEUM & MINERALS**
**DEPARTMENT OF MATHEMATICS & STATISTICS**
**DHAHRAN, SAUDI ARABIA**

STAT 319: PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS

First Major Exam, Semester 063 Solutions
Saturday July 21, 2007

**Instructor**: Marwan Al-Momani **Duration**: 90 Minutes

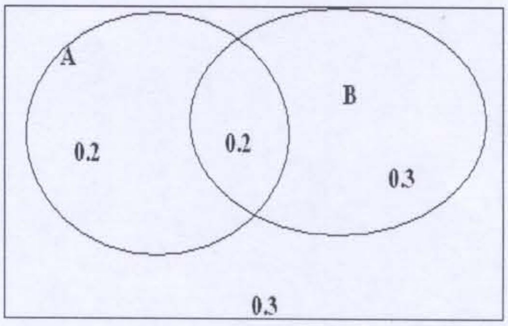
Surname: ID# Serial:

| Question No | Full Marks | Marks Obtained |
|---|---|---|
| **Q1** | **5** | |
| **Q2** | **6** | |
| **Q3** | **15** | |
| **Q4** | **12** | |
| **Q5** | **6** | |
| **Q6** | **8** | |
| **Q7** | **10** | |
| **Q8** | **8** | |
| **Total** | **70** | |

**Question1. (5-Points)** Fill in the blank

1. The sample space is the list of all possible outcomes of a random experiment.
2. The standard deviation of the grades of 5 students is **0**. This implies that all the grades are equal
3. Two events A and B are mutually exclusive if they have no elements in common.
4. A discrete random variable can assume any value in Countable set of numbers.
5. The empirical rule is satisfied when the distribution is bell-shape. . ① pt each.

**Question2.(3+3=6--Points)**
The Venn diagram represents a sample Space **S** and two events **A** and **B**:




**a.** Find $P\left(A\middle|\overline{B}\right)$

$$p(A|B') = \frac{p(A\cap B')}{p(B')}$$ ③ pts

$$= \frac{p(A-B)}{1-p(B)}$$

$$= \frac{p(A) - p(A\cap B)}{1-p(B)} = \frac{.4 - .2}{1-.5} = \frac{0.2}{0.5} = 0.4$$

**b.** Are the two events A and B independent? Explain.

Yes, they are indep. ① pt

because: $p(A) = .4$, $p(B) = .5$, $p(A\cap B) = .2$

$p(A)\cdot p(B) = (.4)(.5) = 0.2 = p(A\cap B)$ ② pts.

**Question 3(4+3+4+4=15-Points)**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 191 | 192 | 193 | 195 | 200 | 200 | 201 | 207 | 208 | 209 |
| 209 | 210 | 212 | 215 | 216 | 224 | 227 | 228 | 230 | 233 |

Given that $\sum x = 4200$, $\sum x^2 = 885298$, answer the following:

**a.** Find the sample mean and standard deviation

I. $\bar{x} = \frac{\sum x_i}{n} = \frac{4200}{20} = 210$ } 1 pt

II. $S = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n-1}}$

$= \sqrt{\frac{885298 - (20)(210)^2}{20-1}}$ } 2 pts

$= \sqrt{\frac{3298}{19}} = 13.1749.$ } 1 pt

**b.** What is the percentage of the observations that lie within two standard devotions from the mean?(2-points)

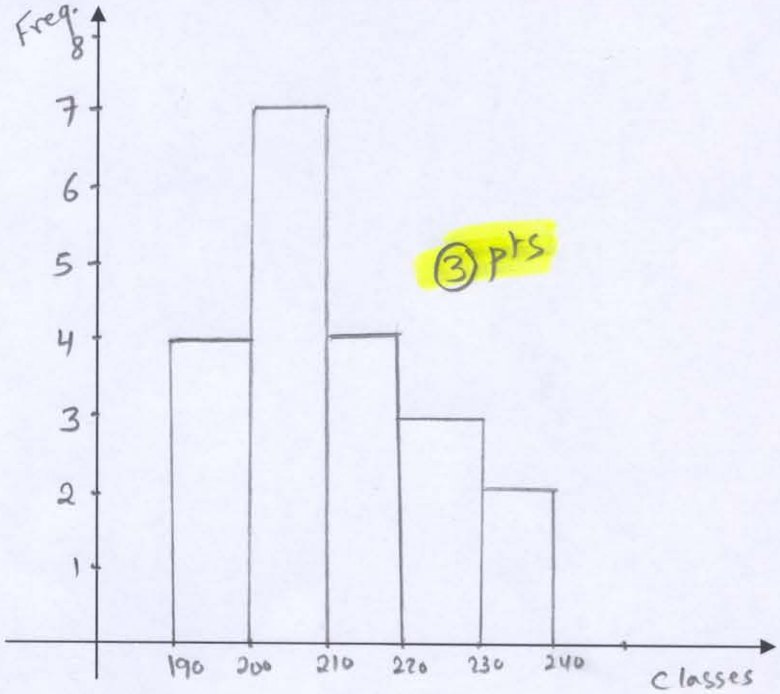
I. $[\bar{x}-2S, \bar{x}+2S] = [210 - 2(13.1749), 210 + 2(13.1749)]$
$= [183.6502, 236.3498]$ } 2 pts

II. All observations lie in this interval $\Rightarrow$ The percentage $= 100\%$ } 1 pt

**c.** Complete the following frequency table, and construct a frequency histogram and comment on the shape.

| Class Interval | Freq. | Relative. Freq. |
|---|---|---|
| 190 ---199 | 4 | 0.20 |
| 200 — 209 | 7 | 0.35 |
| 210 — 219 | 4 | 0.20 |
| 220 — 229 | 3 | 0.15 |
| 230 — 239 | 2 | 0.10 |
| Total | 20 | 1.00 |

4 pts




3 pts

Comment: Freq. histogram is right skewed. 1 pt

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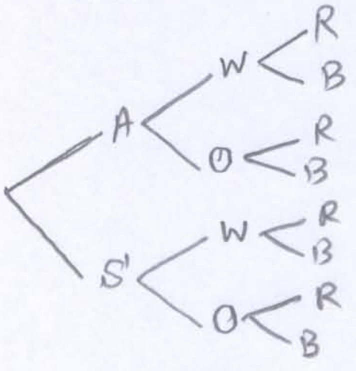
**Question4. (4+4+4=12-Points)**

An order for an automobile can specify either an automatic or a standard transmission, either with or without air-conditioning and any one of the two colors red or blue.

**a.** Write the sample space using the following letters **A**: an automatic automobile, **S**: standard transmission, **W**: With air- conditioning, **O**: without air-conditioning, **R**: Red color, **B**: ~~Black~~ Blue color.





(4) pts

$S = \{ AWR, AWB, AOR, AOB, SWR, SWB, SOR, SOB \}$

**b.** What is the probability of getting an automatic automobile or without air-conditioning system?

Let $E_1$: An automatic automobile

$E_2$: " " without air-conditioning system

$p(E_1 \text{ or } E_2) = p(E_1 \cup E_2) = ?$

$E_1 = \{ AWR, AWB, AOR, AOB \} \rightarrow p(E_1) = \frac{\#(E_1)}{\#(S)} = \frac{4}{8} = \frac{1}{2}$ } (1) pt

$E_2 = \{ AOR, AOB, SOR, SOB \} \rightarrow p(E_2) = \frac{4}{8} = \frac{1}{2}$ } (1) pt

$E_1 \cap E_2 = \{ AOR, AOB \} \rightarrow p(E_1 \cap E_2) = \frac{2}{8} = \frac{1}{4}$. } (1) pt

$\therefore p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$. } (1) pt

**c.** If the color of the automobile is blue, what is the probability that it is with air- conditioning system?

Let $F_1$: The color of the autombile is blue

$F_2$: " automobile with air-conditioning system

$F_1 = \{ AWB, AOB, SWB, SOB \} \rightarrow p(F_1) = \frac{4}{8} = \frac{1}{2}$ } (1) pt

$F_2 = \{ AWR, AWB, SWR, SWB \} \rightarrow F_1 \cap F_2 = \{ AWB, SWB \}$ } (1) pt

$p(F_1 \cap F_2) = \frac{2}{8} = \frac{1}{4}$.

$$p(F_2 \mid F_1) = \frac{p(F_1 \cap F_2)}{p(F_1)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{4} \cdot \frac{2}{1} = \frac{1}{2}.$$ } (2) pts

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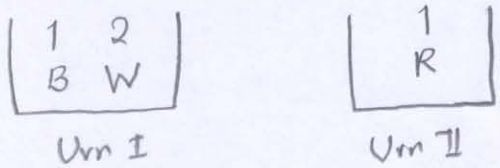
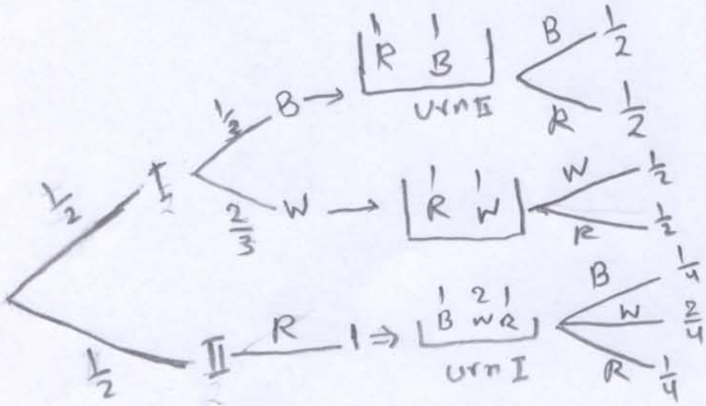
**Question5. (6-Points)**

Urn **I** contains one black and two white balls, and urn **II** contains one red ball. An urn is selected at random, then a ball is randomly drawn from it and placed in the other urn. A ball is then randomly drawn from that urn. Find the probability that this ball is red. (**Hint**: Use the tree diagram)




B: Black ball
W: white ball
R: Red ball




3 pts

$$p(R) = p(R \cap B \cap I) + p(R \cap W \cap I) + p(R \cap R \cap II)$$

$$= \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{1}{2} + \frac{1}{4}\cdot 1\cdot\frac{1}{2}$$

$$= \frac{1}{12} + \frac{2}{12} + \frac{1}{8}$$

$$= \frac{2+4+3}{24} = \frac{9}{24} = \frac{3}{8}$$

$$= 0.375$$

3 pts

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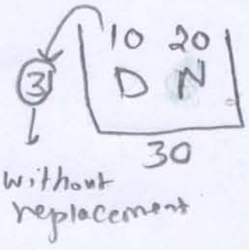
**Question6. (3+3+2= 8-Points)**

A lot of 30 color TV tubes of which 10 are defectives is subjected to an acceptance sampling procedure. **Three** tubes are selected at random, **without replacement.**

a. What is the probability that at least one tube is defective?

let D: The TV tube is defective.
N: " " " " not defective.




$$p(\text{At least one}) = p(X \geq 1) = 1 - p(X<1) = 1 - p(X=0)$$

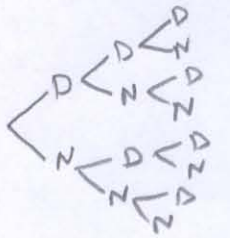
3 pts

$$= 1 - \frac{\binom{10}{0}\binom{20}{3}}{\binom{30}{3}} = 1 - \frac{1140}{4060} = \frac{146}{203} = 0.7192$$

OR: $p(X \geq 1) = p(1) + p(2) + p(3) = \frac{\binom{10}{1}\binom{20}{2}}{\binom{30}{3}} + \frac{\binom{10}{2}\binom{20}{1}}{\binom{30}{3}} + \frac{\binom{10}{3}\binom{20}{0}}{\binom{30}{3}}$

$$= \frac{95}{203} + \frac{45}{203} + \frac{6}{203} = \frac{146}{203} = 0.7192$$

b. What is the probability that the tube on the second draw is defective?




let A: The second tube is defective.

3 pts

$A = \{DDD, DDN, NDD, NDN\}$

$$p(A) = p(DDD) + p(DDN) + p(NDD) + p(NDN)$$

$$= \frac{10}{30}\cdot\frac{9}{29}\cdot\frac{8}{28} + \frac{10}{30}\cdot\frac{9}{29}\cdot\frac{20}{28} + \frac{20}{30}\cdot\frac{10}{29}\cdot\frac{9}{28} + \frac{20}{30}\cdot\frac{10}{29}\cdot\frac{19}{28}$$

$$= \frac{6}{203} + \frac{15}{203} + \frac{15}{203} + \frac{95}{609} = \frac{1}{3} = 0.3333$$

c. What is the probability for part **a** if we sampling **with replacement**?

$$p(X \geq 1) = 1 - p(X=0) = 1 - p(NNN)$$

2 pts

$$= 1 - \left(\frac{20}{30}\right)\left(\frac{20}{30}\right)\left(\frac{20}{30}\right)$$

$$= 1 - \frac{8}{27} = \frac{19}{27} = 0.7037.$$

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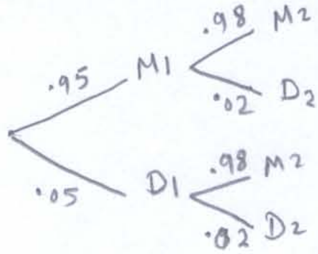
**Question7. (7+ 3= 10-Points)**

An assembly consists of two mechanical components. Suppose that the probabilities that the first and second components meet specifications are **0.95** and **0.98** respectively. Assume that the components are independent. Let X be a random variable represents the number of components in the assembly that meet specification

**a.** Determine the probability distribution of X.
**(Hint:** Use **M1**: The first component meets specification, **D1**: The first component doesn't meet specification, **M2**: The second component meets specification, **D2**: The second component doesn't meet specification)




$$S = \{\underbrace{M1M2}_{2}, \underbrace{M1D2}_{1}, \underbrace{D1M2}_{1}, \underbrace{D1D2}_{0}\}$$

Values of $X$: 0, 1, 2. } 2 pts

$f(0) = p(X=0) = p(D_1D_2) = (.05)(.02) = 0.001$ } 1 pt

$f(1) = p(X=1) = p(M_1D_2) + p(D_1M_2)$
$= (.95)(.02) + (.05)(.98)$
$= 0.019 + 0.049 = 0.068$ } 2 pts

$f(2) = p(X=2) = p(M_1M_2)$
$= (.95)(.98) = 0.931$ } 1 pt

The prob. dist. of X is: } 1 pt

| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 0.001 | .068 | 0.931 |

**b.** Find the expected value of $g(X) = (X-1)^2$

$E(g(X)) = \sum_x g(x) f(x)$
$= \sum (x-1)^2 f(x)$
$= (0-1)^2(.001) + (1-1)^2(.068) + (2-1)^2(.931)$ } 2 pts
$= 0.001 + 0 + 0.931$
$= 0.932$ } 1 pt

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**Question8. (3+3+2= 8-Points)**

Assume the length X in minutes of a particular type of telephone conversation is random variable with probability density function given by:

$$f(x) = \begin{cases} k e^{-x/5}, & x > 0 \\ 0, & otherwise \end{cases}$$

**a.** Find the value of $k$

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1 \Rightarrow \int_0^{\infty} k e^{-x/5}\,dx = -5k\, e^{-x/5}\Big|_0^{\infty} = 1$$

$$-5k(0-1) = 1 \Rightarrow 5k = 1$$

$$\Rightarrow k = \frac{1}{5}$$

③ pts

$$\therefore f(x) = \begin{cases} \frac{1}{5} e^{-x/5}, & x > 0 \\ 0, & o.w \end{cases}$$

**b.** Find the distribution function of the random variable X.

$$F(x) = p(X \le x)$$

$$= \int_{-\infty}^{x} f(t)\,dt = \int_0^{x} \frac{1}{5} e^{-t/5}\,dt$$

$$= -e^{-t/5}\Big|_0^{x} = -e^{-x/5} - (-1)$$

$$= 1 - e^{-x/5}$$

$$\Rightarrow F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1 - e^{-x/5}, & x \ge 0 \end{cases}$$

③ pts

**c.** Find $P(2 < X \le 5)$

$$p(2 < X \le 5) = \int_2^5 \frac{1}{5} e^{-x/5}\,dx = -e^{-x/5}\Big|_2^5$$

$$= -e^{-1} - (-e^{-2/5})$$

$$= e^{-.4} - e^{-1}$$

$$= 0.6703 - 0.3679 = 0.3024$$

② pts

OR $p(2 \le X \le 5) = F(5) - F(2)$

$$= 1 - e^{-1} - (1 - e^{-.4})$$

$$= e^{-.4} - e^{-1} = 0.3024$$