Q1. (7+4+10+2+3 points) ✱SOLUTIONS✱

The data in the following table show the number of Shares Selling (in millions) and the Expected Price (in \$) for 10 selected initial public stock offerings:

Number of Shares Selling (X)	5	9	6.7	8.75	3	13.6	4.6	6.7	3	7.7
Expected Price (Y)	15	14	15	17	11	19	13	14	10	13

Given the sums $\sum x = 68.05$, $\sum y = 141$, $\sum(x-\bar{x})^2 = 92.6722$, $\sum(y-\bar{y})^2 = 62.9$, and $\sum(x-\bar{x})(y-\bar{y}) = 65.8450$, answer the following:

a. Fit the regression equation to predict the Expected Price using the Number of Shares Selling and **interpret** each of the regression coefficients.

1 pt

$$b_1 = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2} = \frac{65.8450}{92.6722} = 0.7105 \quad \text{1 pt}$$

$$b_0 = \bar{y} - b_1\bar{x} = \frac{141}{10} - (.7105)\left(\frac{68.05}{10}\right) \Big\} \text{1 pt}$$

$$= 14.1 - 4.8351$$

$$= 9.2650 \quad \Big\} \text{1 pt}$$

∴ The regression equation is:

$$\hat{y} = 9.2650 + 0.7105X$$

Intrepretation:

1. $b_1 = 0.7105$: As X (Number of shares Selling) increases by one unit (1 million), then Y (Expected Price) increases by 0.7105 units (in \$). 1 pt

2. $b_0 = 9.2650$: when X (Number of Shares Selling) is zero, then the average value of Y (Expected price) is 9.2649 units (in \$). 1 pt

b. Calculate the correlation coefficient between X & Y and **interpret** its value.

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2 \cdot \Sigma(y-\bar{y})^2}} = \frac{65.8450}{\sqrt{(92.6722)(62.9)}} = 0.8624 \; \Big\} \text{2 pt}$$

Interpretation: There is a strong positive linear relationship between X (Number of shares Selling) and Y (Expected price) } 2 pt

c. Using your findings in part (b), and based on 10% significance level, is there a significant linear relationship between the Shares Selling and the Expected Price?

The hypotheses: $H_0: \rho = 0$	$H_A: \rho \neq 0$ ② pts
The assumptions: 1. The data are interval - or ratio - level 2. The two variables X & Y are distributed bivariate normal dist	② pts
The test statistic: $t_c = \dfrac{r}{\sqrt{\dfrac{1-r^2}{n-2}}} = \dfrac{0.8624}{\sqrt{\dfrac{1-(.8624)^2}{10-2}}} = 4.8185$ ① pt ① pt	
The critical value: $t_{\alpha/2, n-2} = t_{.05, 8} = 1.8595$ ① pt	
The decision rule & decision: Reject H_0 if $\lvert t_c \rvert > t_{\alpha/2, n-2}$ ① pt Because $t_c = \lvert 4.821 \rvert > 1.8595$ ✓ } ① pt ∴ Reject H_0	
The conclusion: The linear relationship between the Shares Selling and the Expected Price is significant. ① pt	

d. Predict the Expected Price for 7.7 million Shares Selling.

$$\hat{y}(7.7) = 9.2650 + 0.7105(7.7) \quad \} \text{ 1 pt}$$

$$= \$14.7359 \quad \} \text{ 1 pt}$$

e. Find a 90% CI for the Expected Price of a Number of Shares Selling equal to 7.7 million.

$1 - \alpha = .90 \Rightarrow \alpha = .10$

$t_{\alpha/2, n-2} = t_{.05, 8} = 1.8595$

A 90% C.I. for $y \mid x_p = 7.7$ is:

$$\hat{y} \pm t_{\alpha/2, n} S_\varepsilon \sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{\Sigma(x - \bar{x})^2}}$$

$$S_\varepsilon = \sqrt{\frac{SSE}{n-2}}$$

$SSE = SST - SSR$

$SSR = b_1 \left(\Sigma (x_i - \bar{x})(y_i - \bar{y}) \right) = (.7105)(65.8450) = 46.7829$

$SST = \Sigma (y_i - \bar{y})^2 = 62.9$

$\therefore SSE = 62.9 - 46.7829 = 16.1171$

$$\therefore S_\varepsilon = \sqrt{\frac{16.1171}{10-2}} = 1.4194 \quad \text{1 pt}$$

$\therefore$ A 90% C.I. for y given $x_p = 7.7$ is:

$$14.7359 \pm (1.8595)(1.4194) \sqrt{1 + \frac{1}{10} + \frac{(7.7 - 6.805)^2}{92.6722}}$$

1 pt

$$14.7359 \pm 2.7791$$

$$[11.9568, 17.5150] \quad \text{1 pt}$$

Q2. (2+3+7+4+2 points) 18 pts

A sociologist was hired by a large city hospital to investigate whether the distance (in miles) between home and work for the employees affect the number of unexcused absences per year for the employees. A sample of 10 employees was chosen, and the following regression equation was found:

$$\hat{Y} = 8.10 - 0.344X,\ s_{b_1} = 0.07761, \text{ and } SSE = 13.301$$

Use the previous information to answer the following:

a. **Interpret** the value of b_1.

As X (the distance) increases by one unit (one mile), then Y (The number of unexcused absences per year) decreases by one unit ② pts

b. Construct a 90% CI for the slope of the regression model.

$1-\alpha = .90 \Rightarrow \alpha = .10 \quad \therefore t_{\alpha/2, n-2} = t_{.05, 8} = 1.8595$ ① pt

A 90% C.I. for β_1 is: $b_1 \pm t_{\alpha/2, n-2}\, s_{b_1}$

$-0.344 \pm (1.8595)(0.07761)$ ← ① pt

$-0.344 \pm 0.1443 \Rightarrow [-0.4883, -0.1997]$ ① pt

c. Using (b) test the significance of the regression model at 10% significance level.

The hypotheses: H_0: $\beta_1 = 0$	H_A: $\beta_1 \neq 0$ ② pts
The residuals assumptions: 1. The model errors are normally distributed ② pts 2. " " " have a constant variance at all levels of the indep. variable	
The decision rule & decision: Reject Ho if $(0) \notin$ 90% C.I. for β_1 } ① pt Decision: The zero value $(0) \notin$ C.I. $= [-.4883, -.1997]$ $\therefore$ Reject Ho ① pt	
The conclusion: The regression model used is significant ① pt	

 SOLUTIONS

d. If the correlation coefficient between X & Y is 0.8432, then find the coefficient of determination and **interpret** its value.

$R^2 = r^2$ (for simple linear regression Model) 1 pt

$R^2 = (.8432)^2 = 0.7110 = 71.10\%$ } 1 pt

Interpretation: 71.10% of the total variation happened to Y (number of unexcused absences per year) is explained by the indep. variable X (distance).

2 pts

e. If (12 , 5) is one of the sample observations, then find the error in estimation using the regression model.

$x = 12, \; y = 5$

$\hat{y}(12)$ = The predicted value

$= 8.10 - .344(12) = 3.972$ 1 pt

The error $= y - \hat{y} = 5 - 3.972 = 1.028$ 1 pt

Q3. (1+2+1+2+2+3+3+2 points)

A sample of computer hardware companies was taken from the *Stock Investor Pro*, to study the effect of the Book Value per share ($X1$ in \$), the Return on Equity per share ($X2$ in %), and the Company Type ($X_3 = \begin{cases} 1 \text{ , Old company} \\ 0 \text{ , New company} \end{cases}$) on the Price per share (Y in \$). Use the Minitab output below to answer the questions that follow:

Regression Analysis: Y versus X1, X2, X3

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The regression equation is
Y = 1.71 + 2.50 X1 + 0.497 X2 + 7.25 X3

Predictor       Coef    SE Coef        T       P     VIF
Constant       1.707      6.412     0.27   0.792
X1            2.4987     0.5283     4.73   0.000     1.0
X2            0.4974     0.1167     4.26   0.000     1.0
X3             7.253      5.772     1.26   0.220     1.0

Analysis of Variance

Source            DF        SS       MS       F       P
Regression         3    8918.2   2972.7   12.55   0.000
Residual Error    26    6157.9    236.8
Total             29   15076.1

Predicted Values for New Observations

New Obs     Fit    SE Fit         95.0% CI              95.0% PI
1         51.84      8.90    (  33.54,   70.14)    (  15.30,   88.39)

Values of Predictors for New Observations

New Obs        X1       X2       X3
1            2.33     74.5     1.00
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a. The standard error of the regression model is : $S_\varepsilon = \sqrt{\frac{SSE}{n-k-1}}$	$S_\varepsilon = \sqrt{\frac{6157.9}{30-3-1}} = 15.3897$ 1 pt
b. If the Book Value = the Return on Equity % = 0 and the Company Type is New, then the predicted average Price = $\hat{y} = 1.71 + 2.50(0) + 0.497(0) + 7.25(0) = 1.71$	1.71 or 1.707 2 pts
c. What is the estimated coefficient of the Company Type?	Coefficient = $b_3 = 7.25$ 1 pt
d. Interpret the coefficient in part (c)	Interpretation: The price per share (in $) will increase by 7.25 dollars for old Companies for the Same Book Value per share and same Return on Equity per share, more the new Companies. 2 pts
e. The coefficient of determination is $R^2 = \frac{SSR}{SST} = \frac{8918.2}{15076.1}$	R^2 = 59.15% or 0.5915 2 pts
f. The adjusted coefficient of determination is $R_A^2 = 1-(1-R^2)\left(\frac{n-1}{n-k-1}\right) = 1-(1-0.5915)\left(\frac{29}{26}\right) = 0.5444$	R^2-adj = 54.44% 2 pts
g. Is any of the three independent variables not significant? Why? Yes: X_3 because the p-value = 0.220 ≮ α = .05 (1 pt, 1 pt)	
h. Is the overall model significant? Why? Yes; because from ANOVA table the p-value = 0.000 < α = .05 (1 pt, 1 pt)	
i. If the Book Value is $2.33, the Return on Equity percentage is 74.5%, and the Company is Old, then a 95% CI for the average Price per share is	[33.54 , 70.14] 2 pts

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