Question 1 :(13 Points)

a. Find the dual of the following linear programming problem (*Don't solve*) (5 points)

Minimize $z = 2x_1 - 3x_2 - 5x_3$

Subject to $x_1 - 2x_2 \geq -1$

$2x_1 + x_2 - x_3 \leq 5$

$x_2 + x_3 \leq 4$

$x_1, x_2, x_3 \geq 0$

Solution: Change constraints to: $x_1 - 2x_2 \geq -1$

$-2x_1 - x_2 + x_3 \geq -5$

$-x_2 - x_3 \geq -4$

$x_1, x_2, x_3 \geq 0$ } (2)

The dual: Maximize $W = -y_1 - 5y_2 - 4y_3$ } (1)

Subject to: $y_1 - 2y_2 \leq 2$

$-2y_1 - y_2 - y_3 \leq -3$

$y_2 - y_3 \leq -5$ } (2)

$y_1, y_2, y_3 \geq 0$

b. At what nominal rate of interest compounded semiannually, will money tripled in 10 years. (3 points)

Solution: Let r be the Semiannual rate

$n = (10)(2) = 20$

$S = P(1+r)^n$

$3P = P(1+r)^{20} \Rightarrow (1+r)^{20} = 3$ } (1)

$1 + r = \sqrt[20]{3}$

$r = \sqrt[20]{3} - 1 = 0.0565$ } (1)

$\therefore$ The nominal rate $= 2r$

$= 2(.0565)$

$= 0.113$ } (1)

or $r = 11.3\%$

c. If a man has a choice of investing a sum of money at *9%* compounded annually or *8.8%* compounded semiannually, which one he has to choose? Why?(3 points)

I. For annual rate

$$r_e = (1+r)^1 - 1 = (1+.09)^1 - 1 = .09 = 9\%$$ } (1)

II. For Semiannal interest : $n=2$

$$r_e = \left(1 + \frac{0.088}{2}\right)^2 - 1$$

$$= 0.089936 = 8.994\%$$ } (1)

So, the man must choose to invest his money annually at 9%. } (1)

d. If *$7500* is invested for four years with an interest rate of $5\frac{1}{2}\%$ compounded continuously, then find the compounded amount and compounded interest. (2 points)

Solution: $P = \$7,500$, $r = 0.055$, $n = 4 = t$

The compounded amount $= S = Pe^{rt}$

$= 7.500\, e^{(.055)(4)}$

$= \$9,345.58$ } (1)

The compounded interest $= S - P$

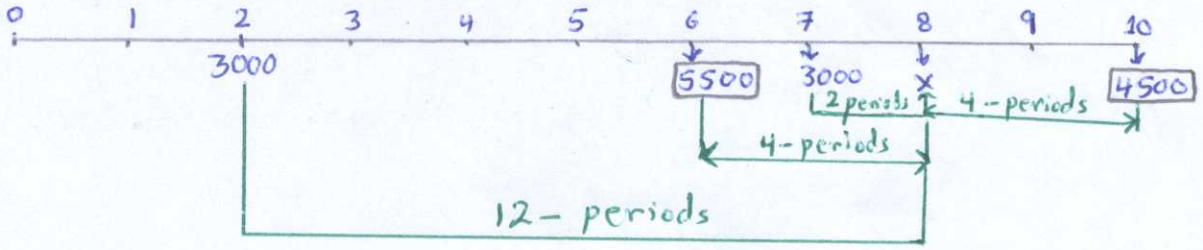
$= 9,345.58 - 7500$

$= \$1,845.58$ } (1)

Question 2 :(10 Points)

a. **A debt of *$5500* due in six years from now and *$4500* due in ten years from now is to be repaid by a payment of *$3000* in two years and *$3000* in seven years and final payment at the end of eight years. If the interest rate is 5% compounded semiannually, how much should the final payment? (6 points)**

Solution: Let X be the final payment, $r = \frac{.05}{2} = 0.025$



(2)

(3) $3000(1+.025)^{12} + 3000(1+.025)^{2} + X = 5500(1+.025)^{4} + 4500(1+.025)^{-4}$

$4034.67 + 3151.875 + X = 6070.971 + 4076.78$

$X = 5500(1.025)^{4} + 4500(1.025)^{-4} - 3000(1.025)^{12} - 3000(1.025)^{2}$

$= 10,147.75 - 7186.545$

$X = \$\ 2,961.21$ } (1)

b. **An initial investment of $25,000 in a business guarantees the following cash flows.**

Year	Cash Flow
3	$8,000
4	$10,000
6	$11,000

If the interest rate is 4% compounded semiannually, determine whether the investment is profitable or not? Why? (4 points)

Solution: $r = \frac{.04}{2} = 0.02$

(2) The net present value = NPV $= 8000(1.02)^{-6} + 10,000(1.02)^{-8} + 11000(1.02)^{-12} - 25000$

$= 7103.771 + 8534.904 + 8693.425 - 25000$

$= 24,312.1 - 25000$

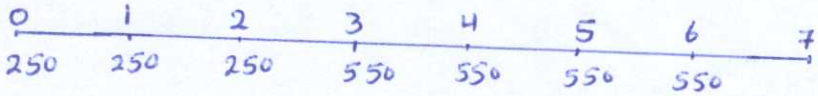
$= -687.9 < 0$ } (1)

The NPV < 0, then it is not profitable } (1)

Question 3 :(7 Points)

a. **Find the present value of an annuity of $250 due at the beginning of each year for three years, and $550 due thereafter at the beginning of each year for four years. If the interest rate is 5% compounded annually?(3 points)**

Solution: Method 1: $r = .05$



The present values of all payments $= 250(1.05)^0 + 250(1.05)^{-1} + 250(1.05)^{-2} + 550(1.05)^{-3} + 550(1.05)^{-4} + 550(1.05)^{-5} + 550(1.05)^{-6}$ } ②

$= 250 + 238.095 + 226.757$

$= \$2483.81$ } ①

—(OR)—

Method 2:

The present values of all payments $= 550(1 + a_{\overline{7-1}|.05}) - 300(1 + a_{\overline{3-1}|.05})$ } ①

$= 550(1 + a_{\overline{6}|.05}) - 300(1 + a_{\overline{2}|.05})$

$\simeq 550(1 + 5.075692) - 300(1 + 1.859414)$ } ①

$\simeq \$2483.81$ } ①

b. **In 12 years a *$42,000* bus will have a salvage value *20%* of its cost. A new bus at that time is expected to sell for *$48,000*. In order to provide funds for the difference between the replacement cost and the salvage value, a sinking fund is setup into which equal payments are placed at the end of each year. If the fund earns *6%* compounded annually, how much should each payment be? (4 points)**

Solution: Time $= n = 12$, $r = .06$, $R = ?$

Salvage Value $= (0.20)(42000) = \$8,400$ } ①

The amount needed after 12 years $= 48000 - 8400$

$= 39,600 = S$ } ①

$S = R \cdot S_{\overline{n}|r} \Rightarrow R = \frac{S}{S_{\overline{n}|r}} = \frac{39600}{S_{\overline{12}|.06}}$ } ①

$\approx \frac{39600}{16.869941} = \$2,347.37$ } ①

—(OR)—

$R = \frac{S}{S_{\overline{n}|r}}$ where $S_{\overline{n}|r} = \frac{(1+r)^n - 1}{r} = \frac{(1.06)^{12} - 1}{.06}$

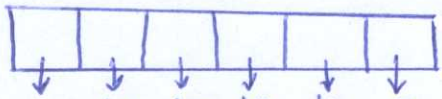
$= 16.869942$

$\therefore R = \frac{39,600}{16.869942} = \$2347.37.$

Question 4 :(6 Points)

a. **Suppose a company wants to make six-digit phone lines, for which the first digit to the left is not zero. How many *even* phone lines does the company have?(2 points)**

Solution:



(Just from 1 to 9) ← 9 | 10 | 10 | 10 | 10 | 5 → (only 0, 2, 4, 6, 8)

(Any number 0 to ... 9)

The number of even phone lines = (9)(10)(10)(10)(10)(5)

= 450,000

②

b. **In how many ways can 8 persons can be distributed to three rooms in a hotel where two rooms with 3 beds, and the third one with 2 beds?(2 points)**

Solution: $n = 8$, $n_1 = 3$, $n_2 = 3$, $n_3 = 2$

$$\text{The number of ways} = \frac{8!}{3!\,3!\,2!}$$

$$= 560$$

②

c. **In a *10-question* examination, each question is graded right or wrong. The student gets *A* grade if he answered at least 8 questions, in how many ways can a student get *A* grade?(2 points)**

Solution:

$$\text{The number of ways} = {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}$$

$$= \frac{10!}{8!\,2!} + \frac{10!}{9!\,1!} + \frac{10!}{10!\,0!}$$

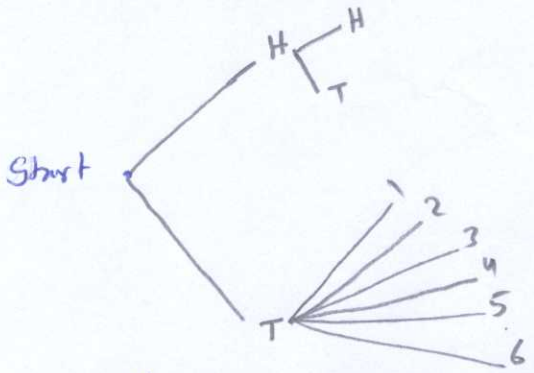
$$= 45 + 10 + 1 = 56$$

②

Question 5: (5 Points)

An experiment consists of flipping a coin and then flipping it a second time if a head occurs. If a tail occurs on the first flip, then a die is tossed once. (Use: H for head, T for tail)

a. **Write the sample space. (1 point)**



$S = \{HH, HT, T1, T2, T3, T4, T5, T6\}$ } (1)

b. **Write the elements of the following events:(1 points each)**
E: **the number appeared on the die is at least two,**
F: **a head occur or the number is odd.**

$E = \{T2, T3, T4, T5, T6\}$ } (1)

$F = \{HH, HT, T1, T3, T5\}$ } (1)

c. **Do the events *E* and *F* are mutually exclusive? Why?(2 points)**

$E \cap F = \{T3, T5\}$ } (1)

$E \cap F \neq \phi \Rightarrow$ E and F are not mutually exclusive } (1)

Question 6 :(9 Points)

A survey was made to study the students' opinion about evening classes. The results are summarized in the following table:

Opinion / Level	Highly agree	Agree	Disagree	Highly disagree	Total
Freshman	45	60	135	65	305
Sophomore	28	15	55	25	123
Junior	17	10	30	15	72
Total	90	85	220	105	500

If one student is selected at random, find the following

a. What is the probability that he student is a junior or disagrees?(3 points)

Let J: The student is junior D: The student disagrees.

$P(J \cup D) = P(J) + P(D) - P(J \cap D)$ } ①

$= \frac{72}{500} + \frac{220}{500} - \frac{30}{500} = \frac{131}{250}$

$= 0.524.$ } ②

b. What are the odds of the event E: the student is freshman and highly agrees?(3 points)

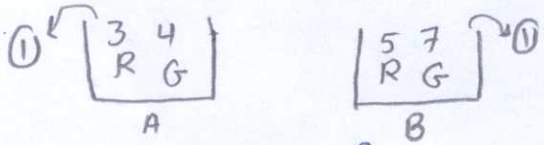
$P(E) = \frac{45}{500} = \frac{9}{100} = .09$ } ①

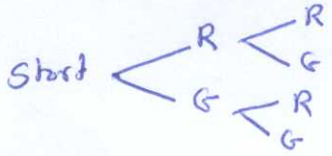
$P(E') = 1 - P(E) = 1 - .09 = 0.91$ } ①

The odds $= \frac{P(E)}{P(E')} = \frac{.09}{.91} = \frac{9}{91}$ or 9:91 } ①

c. Two boxes A and B, if A contains three red and four green balls, B contains five red and seven green balls. One ball is selected at random from each box,

I. Write the sample space.(1 point)





$\Rightarrow S = \{RR, RG, GR, GG\}$ } ①

II. What is the probability of getting one red ball and one green ball? (2 points)

Let $E = \{$ One red ball and one green ball $\}$

$= \{RG, GR\}$ } ①

$P(E) = P(RG) + P(GR)$

$= \left(\frac{3}{7}\right)\left(\frac{7}{12}\right) + \left(\frac{4}{7}\right)\left(\frac{5}{12}\right)$

$= \frac{1}{4} + \frac{5}{21} = \frac{41}{84}$

$= 0.4881$ } ①