Question 1 :(5 Points)

If man has twice as much money invested in bonds paying 10% as he does in stocks paying 12%. Find the total amount he has invested if the total annual interest income from both is \$8,640.

Let X be the amount of money invested in bonds

y " " " " " " stocks

$x = 2y \cdots (1)$, $(.1)x + .12y = 8640 \cdots (2)$ } (2)

By Substituting X from (1) in (2) $\Rightarrow (.1)(2y) + .12y = 8640 \Rightarrow 0.32y = 8640 \Rightarrow y = \frac{8640}{.32}$

$y = 27,000$ } (2)

$\Rightarrow x = 2y = 2(27,000) = 54,000$

$\therefore$ The total amount $= 54,000 + 27,000 = 81,000$ } (1)

Question 2 :(5 Points)

Company A rents cars for \$6 a day and \$0.14 for every mile driven. Company B rents cars for \$12 a day and \$0.08 for every mile driven. If you want to rent a car for 5 days. What is the maximum number of miles that you can drive a Company A car during the f days if it is to cost less than a Company B car?

Let m be the number of miles

Cost of a Company A < Cost of a Company B

$5(6) + .14m < 5(12) + .08m$ } (2)

$30 + .14m < 60 + .08m$

$0.14m - .08m < 60 - 30 \Rightarrow 0.06m < 30 \Rightarrow m < \frac{30}{0.06}$ } (2)

$\Rightarrow m < 500$ miles.

The maximum number of miles is 499 } (1)

Question 3 :(10 Points)

a) Find the equation of the line that passing through (3, -4) and perpendicular to the line $y = 3 + 2x$.(4 Points)

The line is perpendicular to $y = 3 + 2x \Rightarrow$ Slope $= \frac{-1}{2}$. (1)

$\therefore$ The equation: $y - y_1 = m(x - x_1)$ } (1)

$y + 4 = \frac{-1}{2}(x - 3) \Rightarrow y = \frac{-1}{2}x + \frac{3}{2} - 4$

$y = -\frac{1}{2}x - \frac{5}{2}$. } (2)

b) The demand function for a manufacturer product is $p = 1100 - 2q$, where p the price in dollars is, q is the units are demanded per month. Find the number of units that maximizes the total revenue and determine this revenue.(6 Points)

The total revenue $= pq$

$= (1100 - 2q)q = 1100q - 2q^2$ } (1)

$a < 0 \Rightarrow$ it has a maximum value at the vertex.

(1) The vertex : $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$\frac{-b}{2a} = \frac{-1100}{2(-2)} = \frac{-1100}{-4} = 275$

$f\left(\frac{-b}{2a}\right) = f(275) = 1100(275) - 2(275)^2 = \$151,250$ } (2)

$\therefore$ The number of units $= 275$ units

The maximum revenue $= \$151,250$ } (2)

Question 4 :(10 Points)

A manufacturer of a certain product sells all that is produced. If the product is sold \$16 per unit, fixed cost is \$10,000 and variable cost \$8 per unit. Then

a) Determine the break-even point (6 Points)

The price = \$16/unit , F.C. = \$10,000 , V.C. = \$8/unit.

Let q be the number of units

At the break-even point ⇒ Total revenue = Total cost. 1

$\Rightarrow$ 1 $16q = 10{,}000 + 8q \Rightarrow 8q = 10{,}000 \Rightarrow q = 1250$. 2

$T.R. = 16(1250) = \$20{,}000 \Rightarrow$ The break-even point $(1250, \$20{,}000)$ } 2

b) Find the level of production at the break-even point if the total cost increases by 5%. (4 Points)

The new $T.C. = (10{,}000 + 8q) + .05(10{,}000 + 8q) = 10{,}500 + 8.4q$. 2

At the break-even point $T.R = T.C$

$\Rightarrow$ 1 $16q = 10{,}500 + 8.4q \Rightarrow \cancel{7.6q = 10{,}500} \Rightarrow q = \frac{10{,}500}{7.6} = 1381.58 \approx 1382$ 1

The level of production ≈ 1382 units.

Question 5 :(10 Points)

Solve the following linear programming problem **geometrically**:

Minimize:

$$Z = 2x + 1.25y$$

Subject to :

$3x + y \geq 9 \Rightarrow Y_1 = -3x + 9$

x	0	3
y	9	0

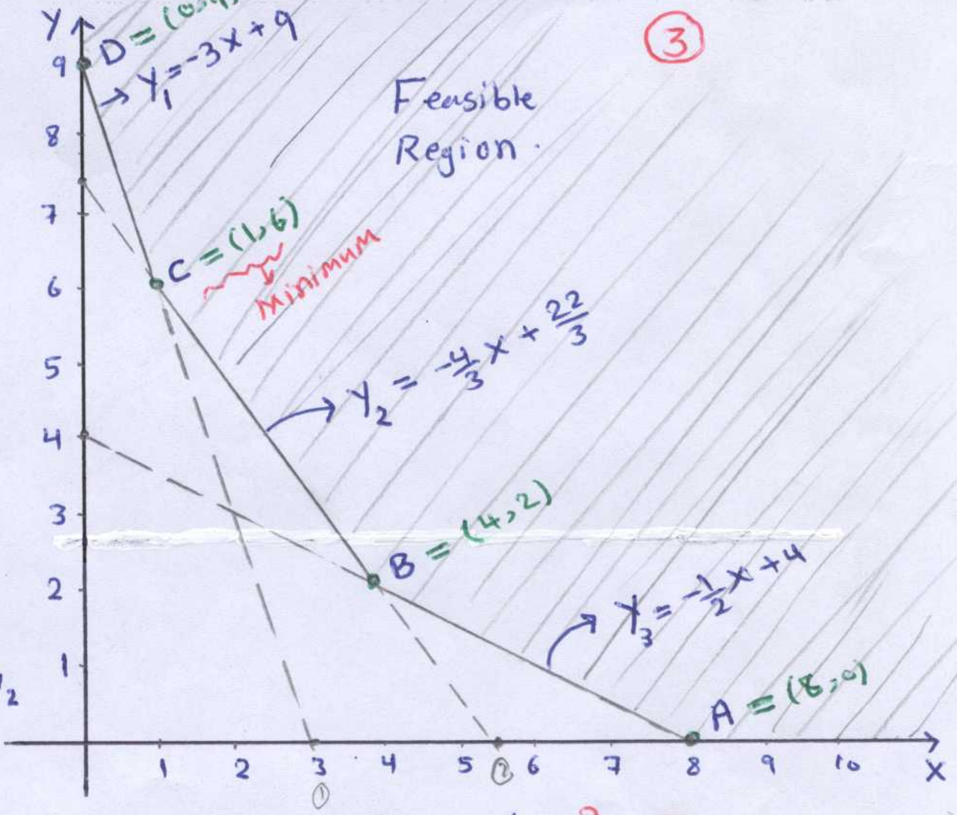
$4x + 3y \geq 22 \Rightarrow Y_2 = -\frac{4}{3}x + \frac{22}{3}$

x	0	11/2
y	22/3	0

$x + 2y \geq 8 \Rightarrow Y_3 = -\frac{1}{2}x + 4$

x	0	8
y	4	0

$x, y \geq 0$



1 $A = (8, 0)$

1 $D = (0, 9)$

B : The intersection point between : Y_2 & Y_3

$-\frac{4}{3}x + \frac{22}{3} = -\frac{1}{2}x + 4$

$-\frac{4}{3}x + \frac{1}{2}x = 4 - \frac{22}{3}$

$-\frac{5}{6}x = -\frac{10}{3}$

$\Rightarrow -15x = -60 \Rightarrow x = 4$

$y = -\frac{1}{2}(4) + 4 = -2 + 4 = 2$

1 $\therefore B = (4, 2)$

C : The intersection between Y_1 & Y_2

$-3x + 9 = -\frac{4}{3}x + \frac{22}{3}$

$-\frac{5}{3}x = -\frac{5}{3} \Rightarrow x = 1, y = 6$

1 $\therefore C = (1, 6) \Rightarrow$

$Z(A) = 2(8) + 1.25(0) = 16 + 0 = 16$

$Z(B) = 2(4) + 1.25(2) = 8 + 2.5 = 10.5$

$Z(C) = 2(1) + 1.25(6) = 2 + 7.5 = 9.5$ → minimum

$Z(D) = 2(0) + 1.25(9) = 0 + 11.25 = 11.25$

} 2

$\therefore$ The minimum value of Z is 9.5 at $C = (1, 6)$ } 1

Question 6:(10 Points)

Use **simplex method** to solve the following linear programming problem:

Maximize:

$$Z = 10x_1 + 15x_2 + 22x_3$$

Subject to :

$$x_1 + 2x_2 + 2x_3 \le 40$$
$$-x_1 - x_2 - 2x_3 \ge -34$$
$$x_1, x_2, x_3 \ge 0$$

Change to : $\Rightarrow x_1 + x_2 + 2x_3 \le 34$. ①

The Standard Form:

$$\left.\begin{aligned} x_1 + 2x_2 + 2x_3 + S_1 &= 40 \\ x_1 + x_2 + 2x_3 + S_2 &= 34 \\ -10x_1 - 15x_2 - 22x_3 + Z &= 0 \end{aligned}\right\} ②$$

Table (1)

entering variable: x_3

	x_1	x_2	x_3	S_1	S_2	Z	b
S_1	1	2	2	1	0	0	40
S_2	1	1	$\boxed{2}$	0	1	0	34
	-10	-15	-22	0	0	1	0

Departing Variable: S_2

Quotients
$40 \div 2 = 20$
$34 \div 2 = 17$ ①

Table (2)

$$\xrightarrow[-R_2+R_1,\ 11R_2+R_3]{\frac{1}{2}R_2}$$

entering variable: x_2

	x_1	x_2	x_3	S_1	S_2	Z	b
S_1	0	$\boxed{1}$	0	1	-1	0	6
x_3	½	½	1	0	½	0	17
Z	1	-4	0	0	11	1	374

Departing Variable: S_1

Quotients
$6 \div 1 = 6$
$17 \div \frac{1}{2} = 34$ ②

Table (3)

$$\xrightarrow[4R_1+R_3]{-\frac{1}{2}R_1+R_2}$$

	x_1	x_2	x_3	S_1	S_2	Z	b
x_2	0	1	0	1	-1	0	6
x_3	½	0	1	-½	1	0	14
Z	1	0	0	4	7	1	398

②

All indicators are non negative.

∴ The maximum value of Z is 398 when
$x_1 = 0$, $x_2 = 6$, $x_3 = 14$. ②