* Solutions *

Question.1. (6-Points)

A person invested \$2500 at an interest rate of 5.5% annually. How much additional money must be invested at an interest rate of 8% annually so that the total interest is 7% of the total amount invested?

Let X be the total amount invested

We have \$2500 invested at 5.5%, then

$(X - 2500)$ invested at 8%

$$\therefore (0.055)(2500) + (0.08)(X - 2500) = 0.07X \quad \} \, 2$$

$$137.5 + 0.08X - 200 = 0.07X \quad \} \, 1$$

$$0.08X - .07X = 200 - 137.5 = 62.5$$

$$\therefore .01X = 62.5 \Rightarrow X = \frac{62.5}{.01}$$

$$\Rightarrow X = 6250 \quad 2$$

$\therefore$ The additional money invested at 8% is: $6250 - 2500 = \$3750$ } 1

Question.2. (6-Points)

A T-shirt manufacturer produces (q) shirts at a total labor cost of $1.3q$ (in dollars) and at a total material cost of $0.4q$ (in dollars). If a fixed cost of \$6500 and each shirt sells for \$3.50, how many must be sold to have a profit more than \$11500.

$$\text{Profit} = T.R. - T.C. > 11500$$

$$T.R. = 3.50q \quad \} \, 1$$

$$T.C. = F.C. + V.C. = 6500 + (1.3q + .4q) = 6500 + 1.7q \quad \} \, 1$$

$$\Rightarrow \text{Profit} = 3.50q - (6500 + 1.7q) > 11500$$

$$1.8q - 6500 > 11500$$

$$\Rightarrow 1.8q > 18000$$

$$\therefore q > \frac{18000}{1.8} = 10000 \quad 3$$

$\therefore$ At least 10001 must be sold to have a profit more than \$11500 } 1

* Solutions *

Question.3. (4+ 4+2 = 10-Points)

a. Find an equation of the line that passes through $(1,9)$ and perpendicular to the line $7y + x = 1$ (write it in slope-intercept form)

$$7y = -x + 1 \Rightarrow y = \frac{-1}{7}x + \frac{1}{7} \quad \} \text{①}$$

The line is perpendicular to $y = \frac{-1}{7}x + \frac{1}{7}$

$$\therefore \quad m = \frac{-1}{\frac{-1}{7}} = 7 \quad \} \text{①} \qquad \begin{matrix} x_1 & y_1 \\ (1, & 9) \end{matrix}$$

The equation:
$$\left.\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 9 &= 7(x - 1) \end{aligned}\right\} \text{①}$$
$$y = 7x - 7 + 9$$

$$y = 7x + 2 \quad \} \text{①}$$

b. The demand function for a publisher's line of cookbooks is $P = 6 - 0.003\,q$ where P is the price per unit when q units are demanded, then:

I. Find the level of production that will maximize the manufacturer's total revenue.

$$\left.\begin{aligned} T.R. = pq &= (6 - .003q)q \\ &= 6q - .003q^2 \end{aligned}\right\} \text{②}$$

$$\left.\begin{aligned} &a = -.003 < 0 \rightarrow \text{it has a maximum value}, \ b = 6 \\ &h = \frac{-b}{2a} = \frac{-6}{2(-.003)} = \frac{6}{.006} = 1000 \end{aligned}\right\} \text{①}$$

$\therefore$ The level of production that maximizes the total revenue $q = 1000$ units. } ①

II. Determine the maximum revenue.

$$\left.\begin{aligned} \text{The maximum revenue} &= f(1000) \\ &= 6(1000) - .003(1000)^2 \\ &= \$\,3000 \end{aligned}\right\} \text{②}$$

* Solutions *

Question.4. (8-Points)

A coffee blend worth \$1.6 per pound is to be mixed with a second coffee blend worth \$3.0 per pound to obtain a mixture worth \$2.40 per pound. How many pounds of each blend should be used in order to obtain 105 pounds of the \$2.40 mixture?

Let x be the amount in pounds of the \$1.6/pound

y = = = = = = = \$3.0/pound

$$x + y = 105 \quad \text{---- (1)} \qquad \} \; 2$$

$$1.6x + 3y = (2.40)(105) = 252 \quad \text{--- (1)} \qquad \} \; 2$$

From (1) $y = 105 - x$ and substitute it in (2)

$$1.6x + 3(105 - x) = 252$$

$$1.6x + 315 - 3x = 252$$

2

$$-1.4x = 252 - 315 = -63$$

$$\therefore x = \frac{-63}{-1.4} = 45 \text{ pounds} \qquad \} \; 1$$

$$\& \; y = 105 - 45 = 60 \text{ pounds} \qquad \} \; 1$$

* Solutions *

Question.5. (2 +4 + 6 = 12-Points)

a. A manufacture of a product sells all that is produced. The total revenue is given by: $T.R. = 7q$ and the total cost $T.C. = 6q + 800$ where q represents the number of units produced and sold.

I. Find the level of production at the break-even point.

At the break-even point.

$$\left.\begin{aligned} T.R. &= T.C. \\ 7q &= 6q + 800 \end{aligned}\right\} 1$$

$$\left.\begin{aligned} 7q - 6q &= 800 \\ \Rightarrow q &= 800 \text{ units} \end{aligned}\right\} 1$$

II. Find the level of production at the break-even point if the total cost increases by 5%.

$$\left.\begin{aligned} \text{The new } T.C. &= \text{Old } T.C. + .05(\text{Old } T.C.) \\ &= (6q + 800) + .05(6q + 800) \\ &= 6.3q + 840 \end{aligned}\right\} 2$$

At break-even point: $T.R. = T.C.$

$$\left.\begin{aligned} 7q &= 6.3q + 840 \\ 7q - 6.3q &= 840 \end{aligned}\right\} 1$$

$$0.7q = 840$$

$$\Rightarrow q = \frac{840}{0.7} = 1200 \text{ units} \Big\} 1$$

b. Solve the non linear system of equations

$$\begin{cases} x\ y = 4 \cdots\cdots\cdots(1) \\ 3y = 2x + 2 \ldots(2) \end{cases}$$

From (1) $y = \frac{4}{x}$ and Substitute it in (2)

$$\left. 3\left(\frac{4}{x}\right) = 2x + 2 \Rightarrow \frac{12}{x} = 2x + 2 \right\} 2$$

$$2x^2 + 2x = 12 \quad (\div 2)$$

$$\left.\begin{aligned} x^2 + x = 6 \Rightarrow x^2 + x - 6 &= 0 \\ (x + 3)(x - 2) &= 0 \\ \Rightarrow x = -3, \quad x &= 2 \end{aligned}\right\} 2$$

I. when $x = -3 \Rightarrow y = \frac{4}{-3} \Rightarrow \left(-3, -\frac{4}{3}\right)$ } 1

II. when $x = 2 \Rightarrow y = \frac{4}{2} = 2 \Rightarrow (2, 2)$ } 1

Solutions

Question.6. (8-Points)

Solve the following system of linear inequalities **geometrically**

$$\begin{cases} 3x - 2y > 6 \\ 2x - 5y \le 10 \end{cases}$$

$\Rightarrow -2y > 6 - 3x \; (\div -2) \Rightarrow y < \frac{3}{2}x - 3 \quad \cdots (1)$ 2

$\Rightarrow -5y \le 10 - 2x \; (\div -5) \Rightarrow y \ge \frac{2}{5}x - 2 \quad \cdots (2)$ 2

(1) $\Rightarrow y = \frac{3}{2}x - 3$

x	0	2
y	-3	0

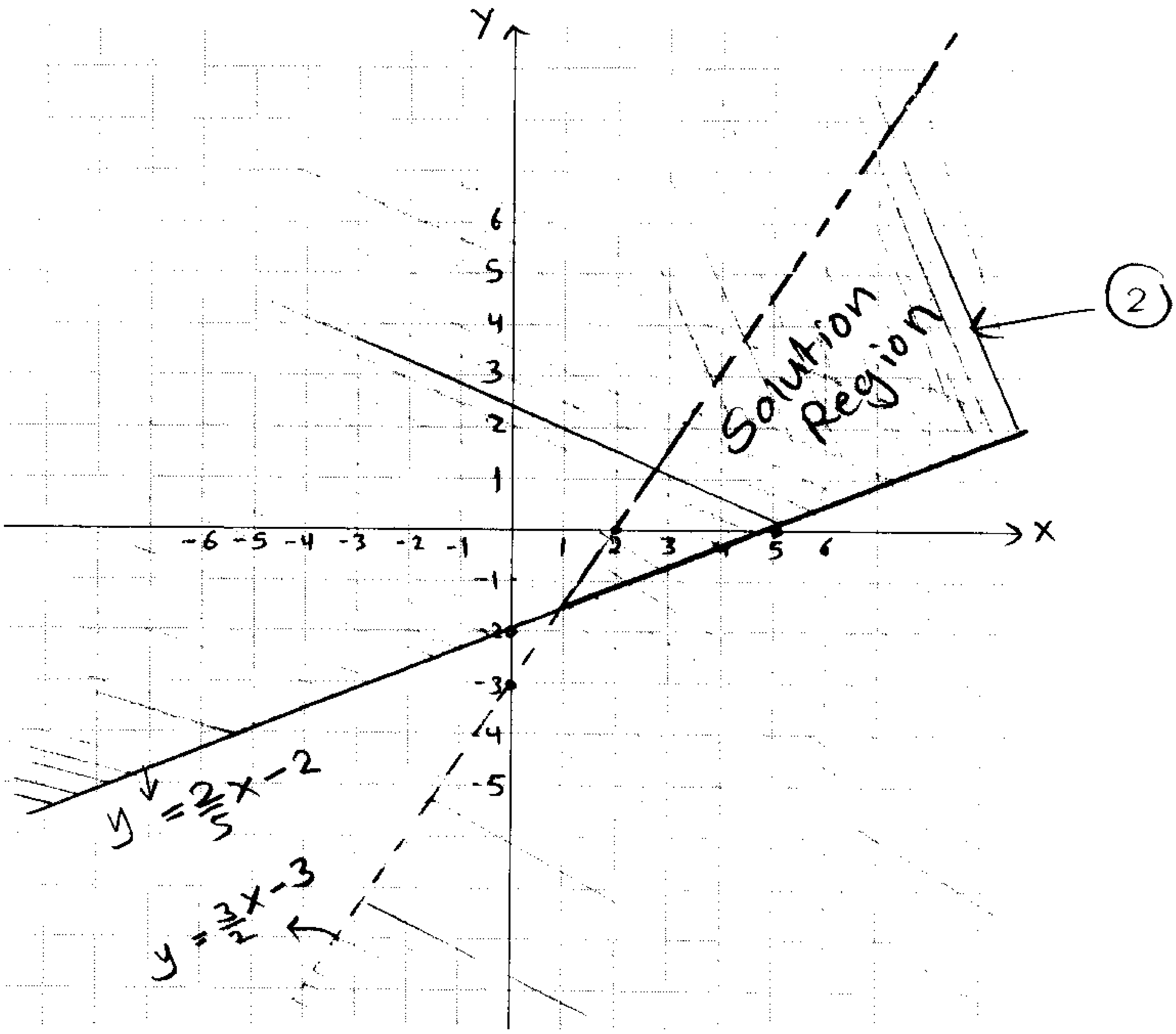
1

(2) $\Rightarrow y = \frac{2}{5}x - 2$

x	0	5
y	-2	0

1

The solution region is the intersection region.



2 For the graph