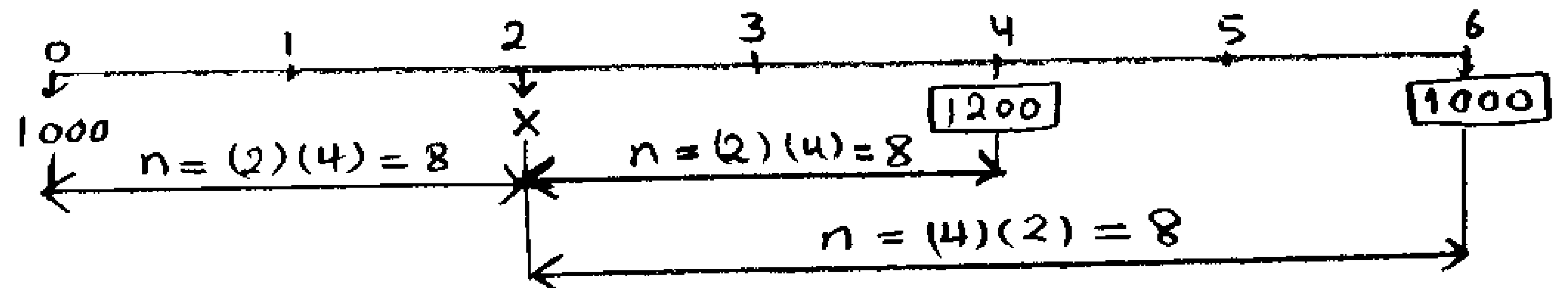
Question.1. (4+5 = 9-Points) * SOLUTIONS *

a. A debt of \$1200 due in four years and \$1000 due in six years is to be repaid by a payment of \$ 1000 now, and a second payment at the end of two years. How much should the second payment be if interest rate is at 8% compounded semiannually?

Let x be the second payment

$r = \frac{.08}{2} = .04$ 1 pt



$$1000(1+.04)^4 + x = 1200(1+.04)^{-4} + 1000(1+.04)^{-8}$$ } 2 pts

$$1169.86 + x = 1025.77 + 730.69$$

$$x = 1756.46 - 1169.86$$

$$x \simeq \$\ 586.60$$

} 1 pt

b. An annuity of equal payments at the end of each quarter for three years is to be purchased of \$ 16000. If the interest rate is 8% compounded quarterly, then how much each payment should be?(Hint: You may use values in the formula sheet)

$r = \frac{.08}{4} = .02$ 1 pt $\qquad n = (3)(4) = 12$ 1 pt

I. $A = R \cdot a_{\overline{n}|r}$

$16000 = R \cdot a_{\overline{12}|.02}$ } 1 pt

$\Rightarrow R = \frac{16000}{a_{\overline{12}|.02}} = \frac{16000}{10.575341} \simeq \$\ 1512.95$ } 1 pt (1 pt)

OR

II. Using the formula:

$$A = R \cdot \frac{1-(1+r)^{-n}}{r}$$

$$16000 = R \cdot \frac{1-(1+.02)^{-12}}{.02}$$ } 1 pt

$$16000 = R \cdot (10.575341)$$ } 1 pt

$$R = \frac{16000}{10.575341} \simeq \$1512.95$$ } 1 pt

SOLUTIONS

Question.2. (4-Points)

Suppose that you invest in a bank account by depositing \$ 2000 at the beginning of every year for the next 15 years. I f the interest rate is 5.7% compounded annually, how much will you have at the end of 15 years?(Hint: Use the formula only)

$r = 5.7\% = 0.057$, $n = 15$, $R = \$2000$

At the begining → Annuity due.

$$\left.\begin{aligned} S &= R \cdot \left[\frac{(1+r)^{n+1} - 1}{r} - 1 \right] \\ &= 2000 \left[\frac{(1+.057)^{16} - 1}{.057} - 1 \right] \end{aligned}\right\} \text{ ② pts}$$

$$= 2000 \left[25.047837 - 1 \right] \quad \} \text{ ① pt}$$

$$= \$\, 48095.67 \quad \} \text{ ① pt}$$

Question.3. (3+ 2 =5-Points)

a. A student must select two courses in the liberal arts and three courses in the social sciences. There are six liberal arts courses and ten social science courses, all of which are different, from which the student may choose. How many selections are possible?

6 liberal arts , 10 Social Science

Number of possible selections $= 6C_2 \cdot 10C_3$ } ② pt

$$\left.\begin{aligned} &= (15)(120) \\ &= 1800 \end{aligned}\right\} \text{ ① pt}$$

b. If the number of possible subsets of a set S equals to 64, then find the number of elements in that set.

Let n be the number of elements in S

$\underbrace{2^n = 64 = 2^6}_{\text{① pt}} \Rightarrow \underbrace{n = 6}_{\text{① pt}}$

SOLUTIONS

Question.4. (3+5=8 -Points)

a. How many four-letter words are possible if the first must be A or D, and the word not ending in letter O. (A word needs not to have a meaning , assume that there are 26 letters)

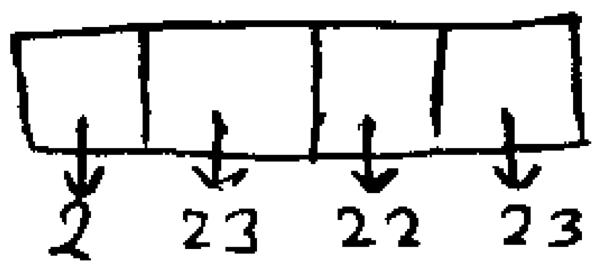
\# of possible words $= (2)(26)(26)(25)$ } 2 pts

$= 33\,800$ } 1 pt

without repetation

\# of words $= (2)(23)(22)(23)$

$= 23\,276$



b. When at least one of four flags colored red, green, yellow, and blue are arranged vertically on a flagpole, the result indicates a signal. How many different signals are possible if at least one flag is used?

\# of possible signals $= {}_4P_1 + {}_4P_2 + {}_4P_3 + {}_4P_4$ } 3 pts

$= \frac{4!}{3!} + \frac{4!}{2!} + \frac{4!}{1!} + \frac{4!}{0!}$ } 1 pt

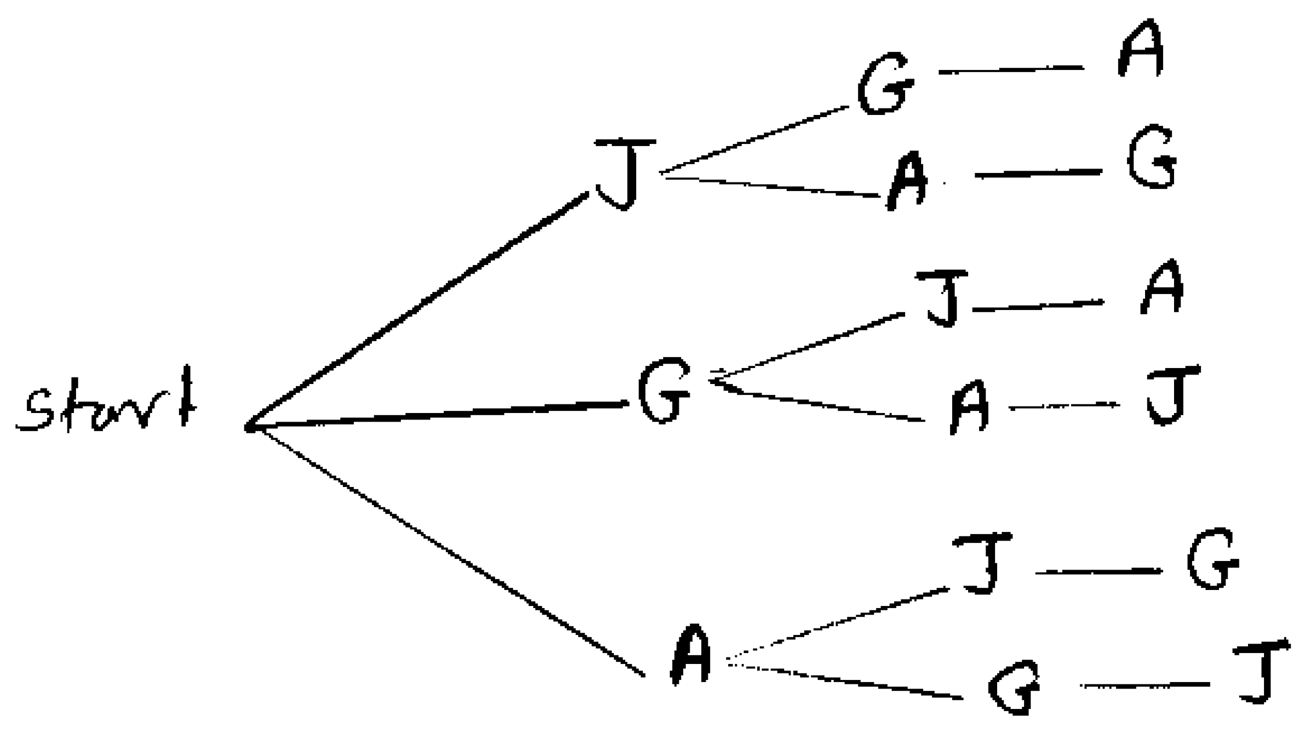
$= 4 + 12 + 24 + 24$

$= 64$ } 1 pt

Question.5. (3+1 +1 + 2 = 7-Points) *SOLUTIONS*

A manger of a company wants to buy three different manufactured cars, one American, one Japanese, and one German. The out come of Japanese (J) car at first, American (A) car at second, and a German (G) car at third can be indicated as JAG. Determine each of the following:

a. The sample space.



③ pts

$S = \{ JGA, JAG, GJA, GAJ, AJG, AGJ \}$

b. If the event E = {Second car is a German Car}, F = {Third car is not an American}. Write the elements of E and F.

$E = \{ JGA, AGJ \}$ ① pt

$F = \{ JAG, GAJ, AJG, AGJ \}$ ① pt

c. Are the two events in part (b) mutually exclusive? Explain why?

① pt NO, because:

$E \cap F = \{ AGJ \} \neq \phi$ ① pt

Question.6. (2+4+2= 8-Points) *SOLUTIONS*

A state legislative body is considering an action that allows gambling within the state. An opinion survey of 200 voters was conducted and the results are indicated in the following table:

	Favor	Oppose	No Opinion	Total
Democrat	40	35	9	84
Republican	60	30	6	96
Other	4	8	8	20
Total	104	73	23	200

Assume that the survey reflects the opinion of the voting population. If a person is selected at random, determine each of the following:

a. The probability that a person opposes gambling within the state.

$$p(\text{oppose}) = \frac{\#(\text{Oppose})}{\#(S)} = \frac{73}{200} = 0.365 \; \Big\} \; ② \text{ pts}$$

b. The probability that the person favors gambling within the states or republican.

F R

$$p(F \cup R) = p(F) + p(R) - p(F \cap R)$$

$$= \frac{104}{200} + \frac{96}{200} - \frac{60}{200} \; \Big\} \; ③ \text{ pts}$$

$$= \frac{140}{200} = 0.70 \; \Big\} \; ① \text{ pt}$$

D

c. If the selected person is Democratic, what is the probability that he or she has no opinion about allowing gambling within the state?

N

$$p(N \mid D) = \frac{\#(N \cap D)}{\#(D)} = \frac{9}{84} = 0.1071 \; \Big\} \; ② \text{ pts}$$

or $$p(N \mid D) = \frac{p(N \cap D)}{p(D)} = \frac{\frac{9}{200}}{\frac{84}{200}} = \frac{9}{84} = 0.1071$$

* SOLUTIONS *

Question.7. (4+4+2= 10-Points)

a. Suppose that 7 cards are randomly drawn without replacement from a deck of 52 cards. What is the probability of getting 2 spades, 3 diamonds?(Hint: use combinations)

Let E : 2 spades, 3 diamons

$$p(E) = \frac{\#(E)}{\#(S)} = \frac{{}_{13}C_2 \cdot {}_{13}C_3 \cdot {}_{26}C_2}{{}_{52}C_7}$$

$$= \frac{(78)(286)(325)}{133\ 784\ 560}$$

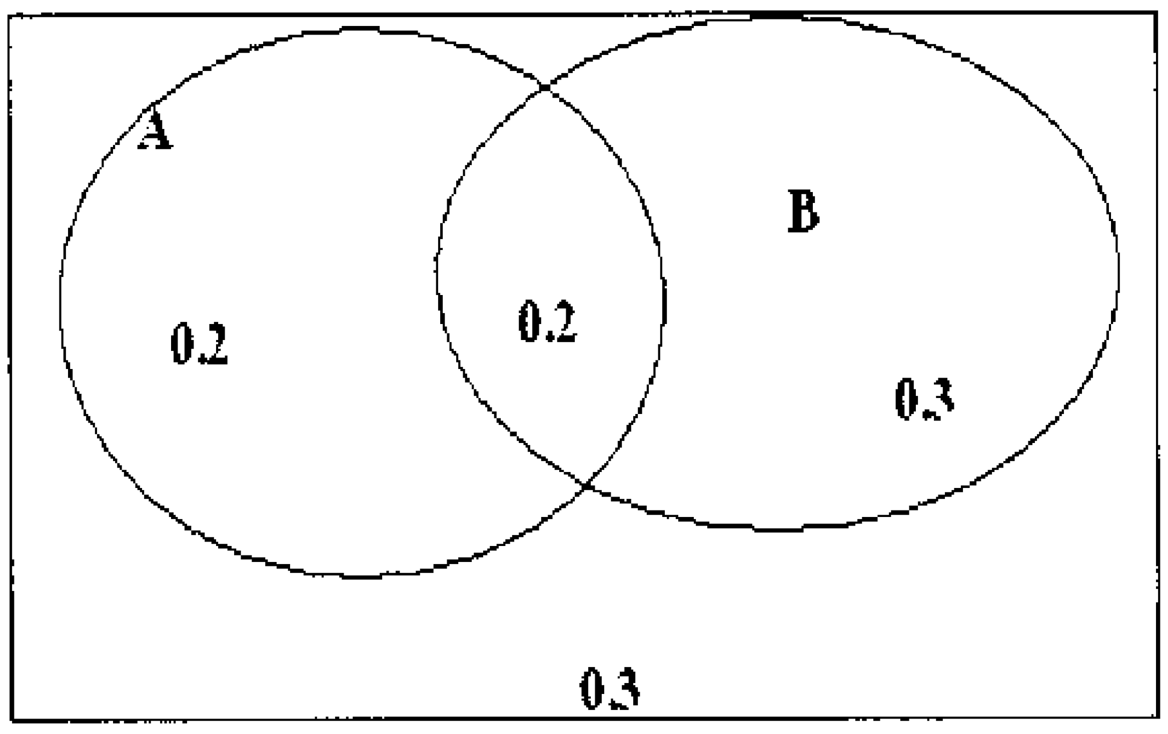
$$= 0.0542$$

(4) pts

b. The Venn diagram represents a sample

Space and two events A and B:

I. Find $P\left(A\middle|\overline{B}\right)$



(1) pt $p(A|B') = \dfrac{p(A \cap B')}{p(B')}$

(2) pts $= \dfrac{0.2}{0.5}$

$= \dfrac{2}{5} = 0.40$ (1) pt

II. Are the two events A and B independent? Explain why?

$p(A) = .4$, $p(B) = .5$, $p(A \cap B) = .2$

$p(A) \cdot p(B) = (.4)(.5) = .20 = p(A \cap B)$

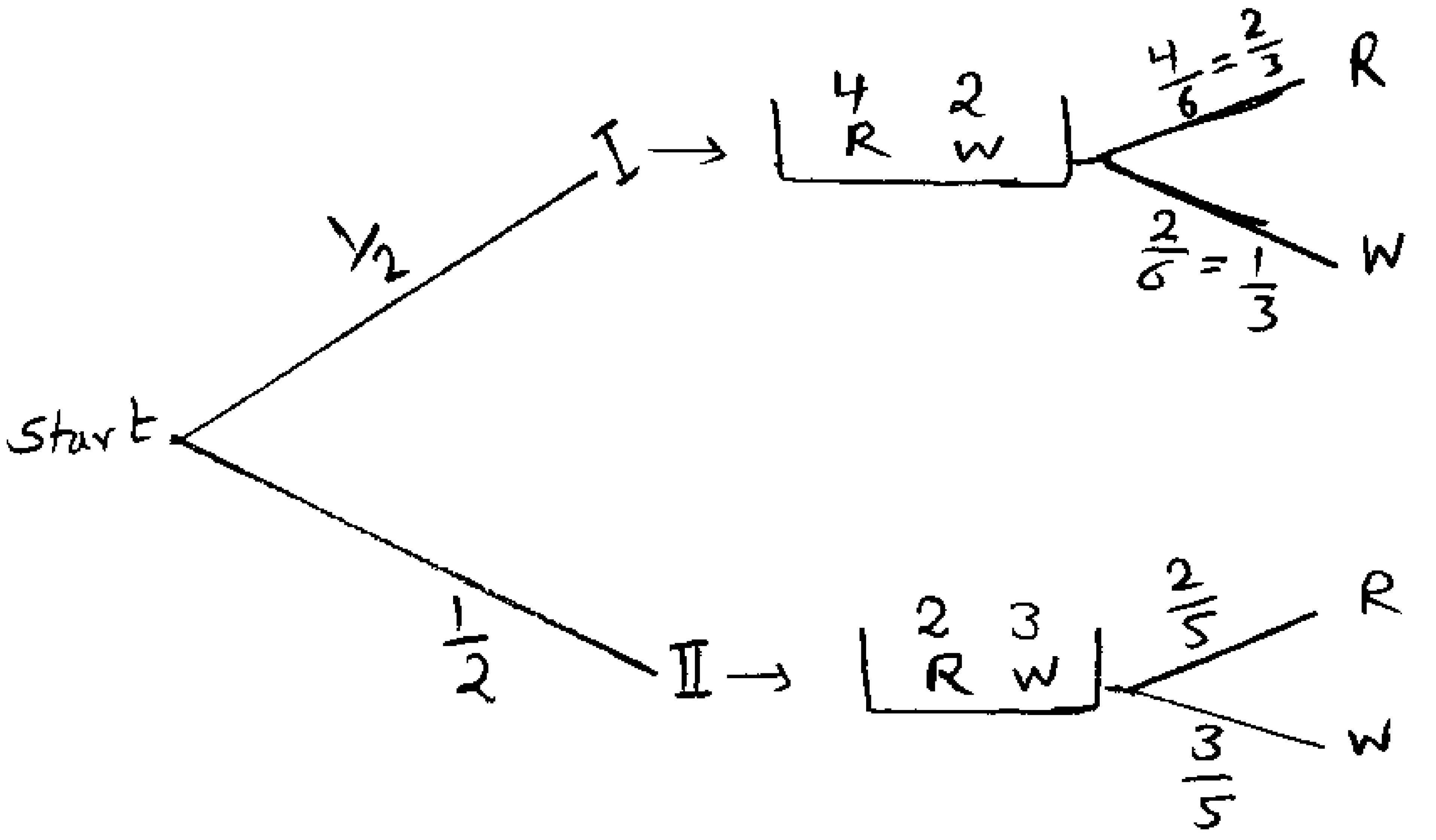
$\therefore$ A & B are indep.

(2) pts

Question.8. (4+ 3+ 2 = 9 -Points) *SOLUTIONS*

Two urns contain marbles, where urn I contains 4 red and 2 white marbles, urn II contains 2 red and 3 white marbles. An urn is chosen at random, and then a marble is randomly drawn from it, then

a. Set the tree diagram with the corresponding probabilities



R : Red Marble
W : white marble

4 pts

b. What is the probability that the marble is white?

$$p(W) = p(W \cap I) + p(W \cap II)$$
$$= p(I) \cdot p(W|I) + p(II) \cdot p(W|II)$$
$$= \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{3}{5} \quad \Big\} \text{ 2 pts}$$
$$= \frac{1}{6} + \frac{3}{10} = \frac{14}{30} = \frac{7}{15} = 0.4667 \quad \text{1 pt}$$

c. If the marble is white, what is the probability that it was drawn from urn II?

$$p(II|W) = \frac{p(II \cap W)}{p(W)} = \frac{p(II) \cdot p(W|II)}{p(W)} = \frac{\frac{1}{2} \cdot \frac{3}{5}}{\frac{7}{15}} \quad \Big\} \text{ 1 pt}$$
$$= \frac{3}{10} \cdot \frac{15}{7} = \frac{45}{70} = \frac{9}{14} \quad \Big\} \text{ 1 pt}$$