* SOLUTIONS *

Question.1. (10-Points)

Solve the following linear programming problem **geometrically**

Minimize $Z = 3x + 4y$

Subject to:
$3x - 4y \le 12, \rightarrow -4y \le -3x + 12 \Rightarrow y \ge \frac{3}{4}x - 3$

$x + 2y \ge 4, \rightarrow 2y \ge -x + 4 \Rightarrow y \ge -\frac{1}{2}x + 2$

$x \ge 1, \rightarrow x \ge 1$

$y \ge 0$

$y = \frac{3}{4}x - 3$

| x | 4 | 8 |
|---|---|---|
| y | 0 | 3 |

$y = -\frac{1}{2}x + 2$

| x | 0 | 4 |
|---|---|---|
| y | 2 | 0 |

$A = (4, 0)$ ① pt

$B \rightarrow$ The intersection point between $x = 1$ & $-\frac{1}{2}x + 2$

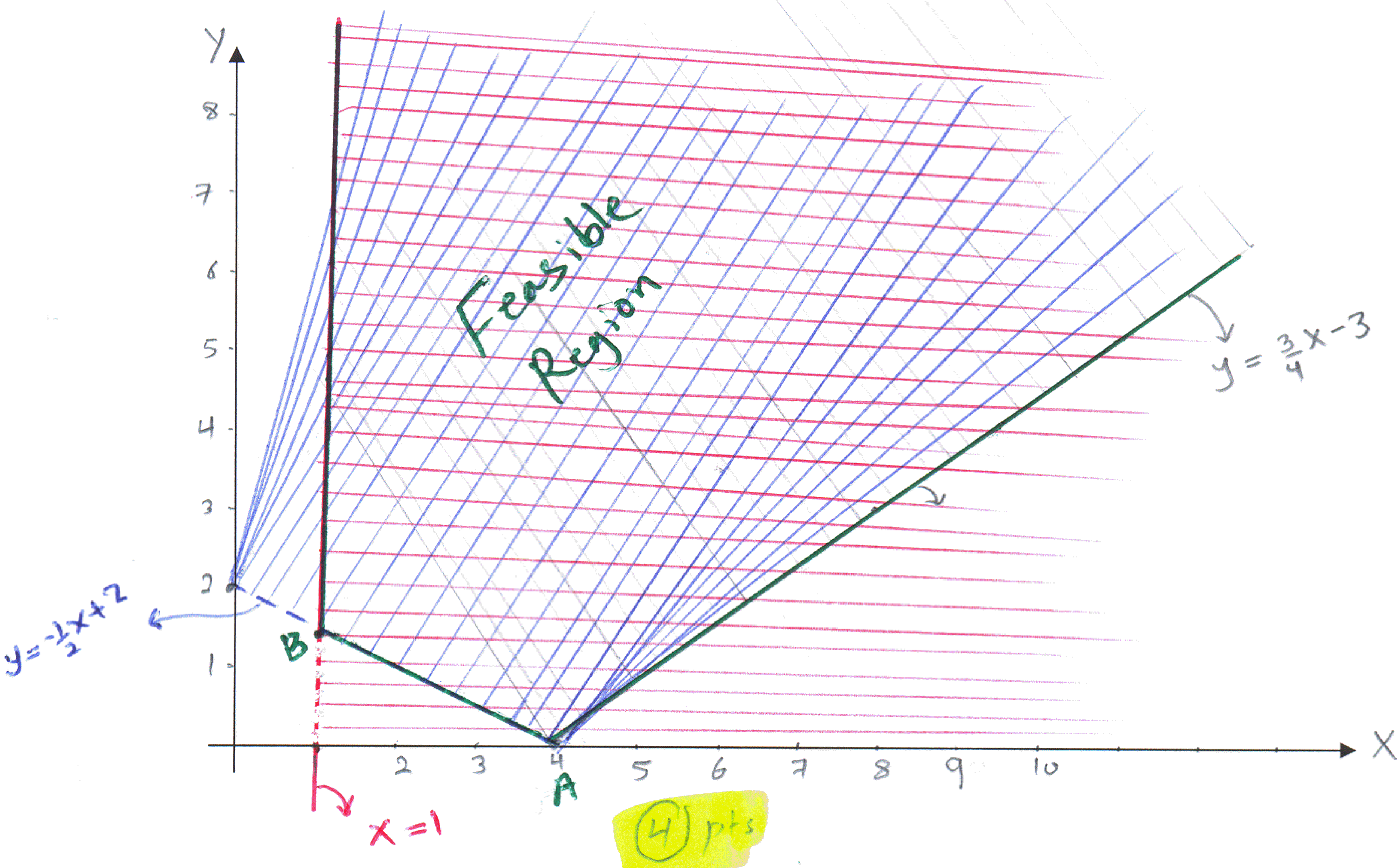
$x = 1, \quad y = -\frac{1}{2}(1) + 2 = -\frac{1}{2} + 2 = \frac{3}{2}$

$\therefore B = (1, \frac{3}{2})$ ① pt

$Z(A) = Z(4, 0) = 3(4) + 4(0) = 12 + 0 = 12$ ① pt

$Z(B) = Z(1, \frac{3}{2}) = 3(1) + 4(\frac{3}{2}) = 3 + 6 = 9$ ① pt

$Z$ has a minimum value at B where $x = 1$, $y = \frac{3}{2}$ and the minimum value is $Z = 9$ } ② pts




④ pts

* SOLUTIONS *

Question.2. (10-Points)
A company manufactures three products X, Y and Z. Each product requires the use of time on machines A, B as given in the following table:

| | Machine A | Machine B | Profit |
|---|---|---|---|
| Product X | 1 hr | 1hr | \$ 10 |
| Product Y | 2hr | 1hr | \$ 15 |
| Product Z | 2hr | 2hr | \$ 22 |

The number of hours per week that A and B are available for production are 40 and 34 hours, respectively. The profit per unit on X, Y and Z is \$10, \$15 and \$22, respectively. What should be the weekly production order if maximum profit is to be obtained using the **simplex method**?

Let $x_1, x_2$ and $x_3$ be the number of units produced weekly from X, Y and Z respectively.
Let P : The profit obtained.

We want to: Maxmize $P = 10x_1 + 15x_2 + 22x_3$
subject to:
$x_1 + 2x_2 + 2x_3 \le 40$
$x_1 + x_2 + 2x_3 \le 34$
$x_1, x_2, x_3 \ge 0$
(2) pts

$x_1 + 2x_2 + 2x_3 + S_1 = 40$
$x_1 + x_2 + 2x_3 + S_2 = 34$
$-10x_1 - 15x_2 - 22x_3 + P = 0$
(1) pt

| | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | P | b |
|---|---|---|---|---|---|---|---|
| $S_1$ | 1 | 2 | 2 | 1 | 0 | 0 | 40 |
| $S_2$ | 1 | 1 | $\boxed{2}$ | 0 | 1 | 0 | 34 |
| P | -10 | -15 | -22 | 0 | 0 | 1 | 0 |

Quotients
$40 \div 2 = 20$
$34 \div 2 = 17$
(2) pts

(1) pt

$\xrightarrow{\frac{1}{2}R_2}$

| | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | P | b |
|---|---|---|---|---|---|---|---|
| $S_1$ | 1 | 2 | 2 | 1 | 0 | 0 | 40 |
| $x_3$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\boxed{1}$ | 0 | $\frac{1}{2}$ | 0 | 17 |
| P | -10 | -15 | -22 | 0 | 0 | 1 | 0 |

$-2R_2+R_1$
$\xrightarrow{22R_2+R_3}$

| | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | P | b | Quotient |
|---|---|---|---|---|---|---|---|---|
| $S_1$ | 0 | $\boxed{1}$ | 0 | 1 | -1 | 0 | 6 | $6 \div 1 = 6$ |
| $x_3$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 1 | 0 | $\frac{1}{2}$ | 0 | 17 | $17 \div \frac{1}{2} = 34$ |
| P | 1 | -4 | 0 | 0 | 11 | 1 | 374 | |

$-\frac{1}{2}R_1 + R_2$
$\xrightarrow{4R_1 + R_3}$

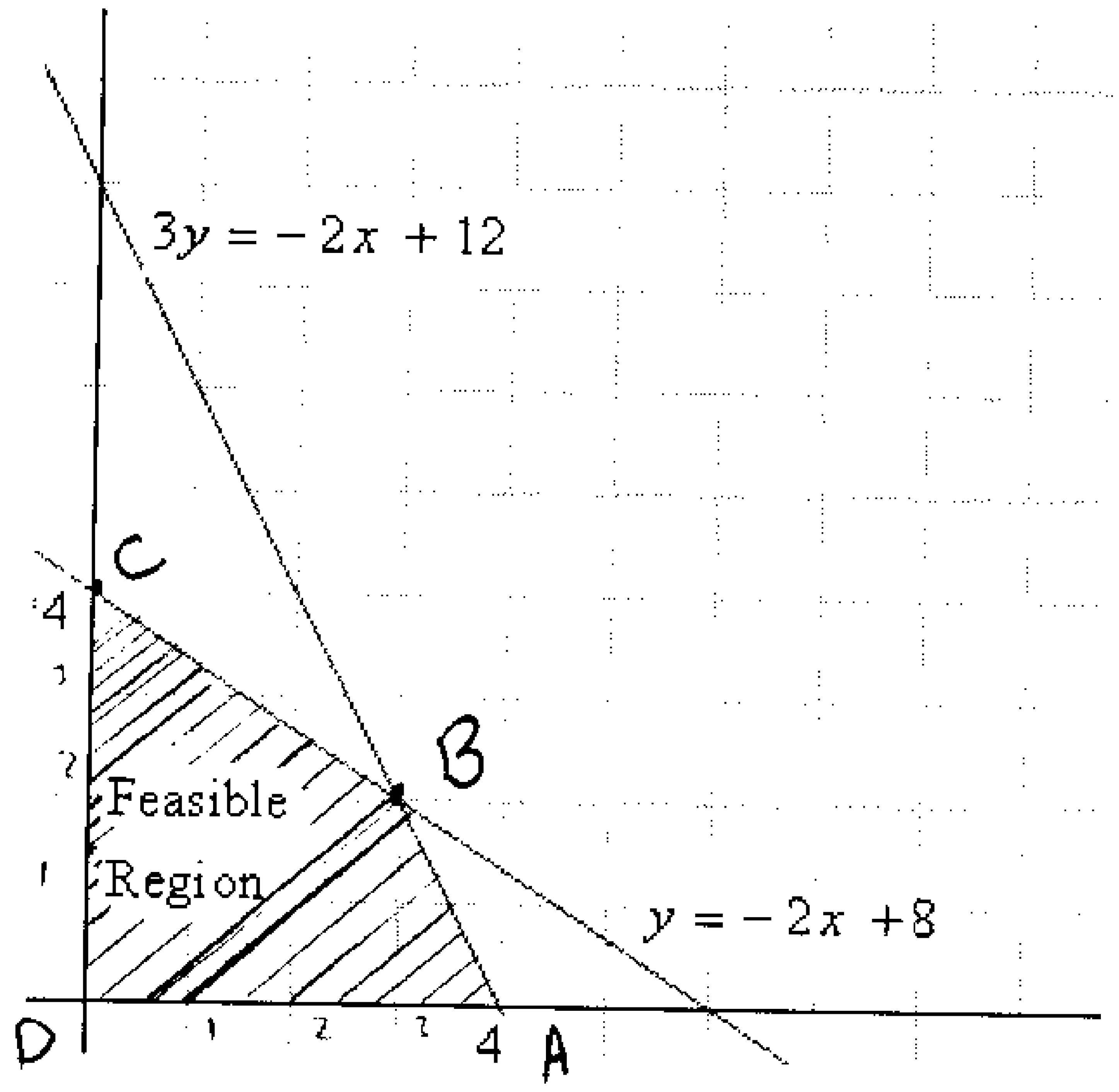
| | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | P | b |
|---|---|---|---|---|---|---|---|
| $x_2$ | 0 | 1 | 0 | 1 | -1 | 0 | 6 |
| $x_3$ | $\frac{1}{2}$ | 0 | 1 | $-\frac{1}{2}$ | 1 | 0 | 14 |
| P | 1 | 0 | 0 | 4 | 7 | 1 | 398 |

(2) pts

The maximum profit is $P = \$ 398$ when $x_1 = 0$, $x_2 = 6$, $x_3 = 14$.
(2) pts

* SOLUTIONS *

Question.3. (8-Points)
Consider the following graph which shows the feasible region for a linear programming problem:




Depending on the on the above graph find the optimum solution(s) that maximizes the following objective function: $Z = 18x + 9y$

Let the intersection points be A, B, C & D as appear on the graph

$A = (4,0)$ , $B = (3,2)$ , $C = (0,4)$ , $D(0,0)$ } ② pts

$Z(A) = Z(4,0) = 18(4) + 9(0) = 72 + 0 = 72$ ①
$Z(B) = Z(3,2) = 18(3) + 9(2) = 54 + 18 = 72$ ① pt
$Z(C) = Z(0,4) = 18(0) + 9(4) = 0 + 36 = 36$ ① pt
$Z(D) = Z(0,0) = 18(0) + 9(0) = 0 + 0 = 0$ ① pt

∴ Z has a maximum values at A,B and so, it has maximum value at all points (x,y) on the line segment A,B, where

$x = (1-t)(4) + t(3) = 4 - 4t + 3t = 4 - t$
$y = (1-t)(0) + t(2) = 0 + 2t = 2t$ , $0 \le t \le 1$ } ② pts

OR

$x = (1-t)(3) + 4t = 3 - 3t + 4t = 3 + t$
$y = (1-t)(2) + t(0) = 2 - 2t + 0 = 2 - 2t$ , $0 \le t \le 1$

* SOLUTIONS *

Question.4. (7+3 =10 -Points)

a. Write the dual for the following linear programming problem(**DO NOT SOLVE**)

Minimize $Z = x_1 + 8x_2 + 5x_3$

Subject to: $x_1 + x_2 + x_3 \geq 8$, $\quad x_1 + x_2 + x_3 \geq 8$

$x_1 - 2x_2 - x_3 \leq -2$, $\Rightarrow$ multiply it by (-1) $\Rightarrow$ $-x_1 + 2x_2 + x_3 \geq 2$ 1 pt

$x_1, x_2, x_3 \geq 0$

The dual is:

1 pt Maxmize: $W = 8y_1 + 2y_2$ } 1 pt

Subject to:

$$\left.\begin{aligned} y_1 - y_2 &\leq 1 \\ y_1 + 2y_2 &\leq 8 \\ y_1 + y_2 &\leq 5 \\ y_1, y_2 &\geq 0 \end{aligned}\right\} \text{(4) pts}$$

b. Consider the following linear programming problem:

Minimize $Z = \cancel{x_1}\,2x_1 + 2x_2$ $\rightarrow$ $Z = 2x_1 + 2x_2$.

Subject to: $x_1 + x_2 \geq 80$,

$3x_1 + 2x_2 \geq 160$,

$5x_1 + 2x_2 \geq 200$

$x_1, x_2 \geq 0$

Suppose after using the simplex method to solve the **dual** for the above problem the last simplex tableau is given by:

| | $y_1$ | $y_2$ | $y_3$ | $s_1$ | $s_2$ | $W$ | $b$ |
|---|---|---|---|---|---|---|---|
| $y_2$ | 0 | 4 | 8 | 2 | - 1 | 0 | 2 |
| $y_1$ | 4 | 0 | - 4 | - 2 | 3 | 0 | 2 |
| $W$ | 0 | 0 | 40 | 40 | 20 | 1 | 120 |

Find the solution for the objective function Z.

The solution for Z is: $x_1 = 40$

$x_2 = 20$

& $Z = 120$ } 3 pts

*SOLUTIONS*

Question.5. (4 + 2 = 6-Points)
Over a six-year period, an original principal of 8000 SR is accumulated to 12867.5 SR in an account in which interest was compounded quarterly. Find:

a. The nominal rate of interest

$P = 8000$ , $S = 12867.5$ , 6 yrs (quarterly) , $r = ?$

Let $r$ be the quarterly rate of interest

$n = (6)(4) = 24$ 1 pt

$$S = P(1+r)^n \Rightarrow 12867.5 = 8000(1+r)^{24}$$

$$\left.\frac{12867.5}{8000} = (1+r)^{24} \right\} \text{1 pt}$$

$$\sqrt[24]{\frac{12867.5}{8000}} = 1+r \Rightarrow \sqrt[24]{1.6084375} = 1+r$$

$$1.02 = 1+r$$

$$\left.\Rightarrow r = 1.02 - 1 = .02 \right\} \text{1 pt}$$

$$\left.\therefore \text{The nominal rate} = 4r = 4(.02) = .08 = 8\% \right\} \text{1 pt}$$

b. The compounded interest.

$$\text{The compounded Interest} = S - P$$
$$\left. = 12867.5 - 8000 \right\} \text{2 pts}$$
$$= 4867.5 \text{ SR}$$

* SOLUTIONS *

Question.6. (3 + 3 = 6-Points)

a. A trust fund is being set up so that at the end of 15 years there will be 100,000 SR. If interest is compounded continuously at an annual rate of 6.5%, how much money (In SR) should be paid into the fund initially?

$t = 15$ , $S = 100,000$ SR , $P = ?$ , $r = 6.5\%$

$$\left.\begin{aligned} P &= S e^{-rt} \\ &= 100,000\, e^{-(.065)(15)} \\ &= 100,000\, e^{-0.975} \end{aligned}\right\} \text{② pts}$$

$$= 37,719.23 \text{ SR} \quad \} \text{ ① pt}$$

b. If interest is compounded continuously at an annual rate of 7%, how many years would it take for a principal P to triple (Becomes three times)? Give your answer to the nearest year

$r = 7\%$ , $t = ?$ $S = 3P$

$S = Pe^{rt} \Rightarrow 3P = P e^{.07t}$ $(\div P)$ } ① pt

$3 = e^{.07t}$ Take ln

$\ln 3 = \ln e^{.07t} \Rightarrow .07t = \ln 3$ } ① pt

$\therefore t = \dfrac{\ln 3}{.07} = 15.6945$ yrs

$\simeq 16$ years } ① pt