SOLUTIONS

King Fahd University of Petroleum & Minerals
Department of Mathematical Science
STAT-212-Term051-I -Quiz #5

Name: ID: Serial:

Question One (5-Points)

State University recently randomly sampled Eight students and analyzed grade point average (GPA-y) and number of study hours per week (x). The following data were observed:

Hours (x)	30	25	28	20	22	10	19	22
GPA (y)	3.45	3.12	3.4	3.22	3	2.8	3	3.1

Given that:

$\sum X = 176, \sum Y = 25.09, \sum XY = 560.3, \sum X^2 = 4138, \sum Y^2 = 79.0153, SSE = \mathbf{0.066553}$

a. Obtain the best fit for the regression line equation

$$b_1 = \frac{\Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} = \frac{560.3 - \frac{(176)(25.09)}{8}}{4138 - \frac{(176)^2}{8}}$$

$$= \frac{8.32}{266} = 0.0313$$

$$b_0 = \bar{y} - b_1\bar{x} = \left(\frac{25.09}{8}\right) - (0.0313)\left(\frac{176}{8}\right) = 2.4477$$

$$\therefore \quad \hat{y} = b_0 + b_1 X$$

$$\hat{y} = 2.4477 + 0.0313X$$

b. Do you think that there is a linear relation between the GPA and the number of study hours per week? test using $\alpha = .05$

1. $H_0: \beta_1 = 0$ vs $H_A: \beta_1 \neq 0$

2. $t_c = \frac{b_1 - 0}{S_{b_1}}$, $S_{b_1} = \frac{S_\varepsilon}{\sqrt{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}}$, $S_\varepsilon = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{0.066553}{6}} = 0.1054$

$$t_c = \frac{0.0313}{0.0065} = 4.8433 \quad \Rightarrow \quad S_{b_1} = \frac{0.1054}{\sqrt{4138 - \frac{(176)^2}{8}}} = 0.0065$$

3. $t_{\alpha/2, n-2} = t_{.025, 6} = 2.4469$

Reject H_0 if $t_c > t_{\alpha/2, n-2} \Rightarrow 4.8433 > 2.4469$

$\therefore$ Reject H_0

4. Conclusion: Based on the sample data, there is a significant linear relation between the GPA and the number of study hours per week.

c. If the correlation coefficient is 89.24%, find the coefficient of determination and interpret its value.

$r = 89.24\% = 0.8924$

$R^2 = r^2$ (For simple linear regression only)

$= (0.8924)^2 = 0.7964$

Interpretation: Number of study hours per week explains 79.64% of the total variation in the GPA.

d. Find a 95% C.I. for the mean GPA given that the number of study hours per week is 24 hours?

$\hat{y}(24) = 2.4477 + .0313(24)$

$= 3.1989$

$t_{\alpha/2, n-2} = t_{.025, 6} = 2.4469$

A 95% C.I for $\mu_{Y|X_p=24}$ is:

$$\hat{y} \pm t_{\alpha/2, n-2} \cdot S_\varepsilon \sqrt{\frac{1}{n} + \frac{(X_p - \bar{X})^2}{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}}$$

$$3.1989 \pm (2.4469)(0.1054)\sqrt{\frac{1}{8} + \frac{(24-22)^2}{266}}$$

3.1989 ± 0.0965

$[3.1024, 3.2954]$.