

1. A tourism and traveling agency claims that the average of a one-day travel expenses in Moscow exceed \$500. If a random sample of 35 one-day travel expenses in Moscow has a mean of \$538 and a standard deviation of \$41, is the claim of the company true? Use the critical value approach and $\alpha = 10\%$.

The hypotheses are: $H_0: \mu \leq 500$ $H_A: \mu > 500$ } ① point

The assumption is: Large Sample size, $n \geq 30$ } ① point

The test statistic: $n = 35$, $\bar{x} = 538$, $s = 41$

$$Z_c = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{538 - 500}{41/\sqrt{35}} = 5.48 \quad \left. \vphantom{\frac{\bar{x} - \mu_0}{s/\sqrt{n}}} \right\} \text{ ② points}$$

$$\bar{x}_{\alpha U} = \mu_0 + Z_{\alpha} \frac{s}{\sqrt{n}} = 500 + (1.28) \frac{41}{\sqrt{35}} = \boxed{508.87}$$

The critical value: $\alpha = .10 \Rightarrow Z_{\alpha} = Z_{.10} = 1.28$ } ① point

Decision Rule: Reject H_0 if $Z_c > Z_{\alpha}$
 $\Rightarrow 5.48 > 1.28$
 \therefore Reject H_0 } ② points

If $\bar{x} > \bar{x}_{\alpha U} \Rightarrow$ Reject H_0 . So, since $\bar{x} = 538 > 508.87 = \bar{x}_{\alpha U} \Rightarrow$ Rej. H_0 .

Conclusion: A tourism and traveling agency claims based on the sample data is true, that means the average of a one-day travel expenses in Moscow exceeds \$500.

① point.

2. Assume that the UK insurance survey is based on 1,000 randomly selected United Kingdom households and that 640 of these households spent on life insurance in 1993. Using the p-value approach test the claim that more than 60% of UK households spent on life insurance in 1993. Use 5% level of significance.

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| <p>The hypotheses are: $H_0: p \leq 0.60$ $H_A: p > 0.60$ (1) point</p> |
| <p>The assumptions are:</p> <p>a. Large Sample Size b. $np \geq 5$ c. $n(1-p) \geq 5$</p> <p>} (2) points</p> |
| <p>The test statistic value:</p> $\bar{p} = \frac{x}{n} = \frac{640}{1000} = 0.64 \quad \text{(1) point}$ $Z_c = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.64 - 0.60}{\sqrt{\frac{(0.6)(1-0.6)}{1000}}} = 2.58 \quad \text{(1) point}$ |
| <p>The p-value = $p(Z > z_c)$ $= p(Z > 2.58)$ $= 0.5 - p(0 < Z < 2.58)$ $= 0.5 - 0.4951$ $= 0.0049$</p> <p>} (2) points</p> |
| <p>Decision Rule: Reject H_0 if p-value $< \alpha$ $\Rightarrow 0.0049 < 0.05$ \therefore Reject H_0</p> <p>} (2) points</p> |
| <p>Conclusion: Based on the sample data there are <u>more than 60%</u> of UK households spent on life insurance in 1993. (1) point</p> |

3. Starting annual salaries for individuals with master's and bachelor's degrees were collected in two different samples. The data are given as follows:

Master's Degree

$n_1 = 25$

$\bar{x}_1 = \$45,000$

$s_1 = \$4,000$

Bachelor's Degree

$n_2 = 25$

$\bar{x}_2 = \$35,000$

$s_2 = \$3,500$

Do the data provide sufficient evidence to conclude that there is no difference between the average annual salaries of the two degrees? Use a significance level of 0.05.

The hypotheses are: $H_0: \mu_1 - \mu_2 = 0$

$H_A: \mu_1 - \mu_2 \neq 0$ ①

The assumptions are: a. Each pop. has a normal dist. b. σ_1^2, σ_2^2 are unknown but equal

② points

c. Samples are indep.

d. Small samples size ($n_1 < 30$ or $n_2 < 30$)

The test statistic value:

$$t_c = \frac{\bar{x}_1 - \bar{x}_2 - 0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \quad S_p = \sqrt{\frac{(25-1)(4000)^2 + (25-1)(3500)^2}{25+25-2}}$$

$$= 3758.32 \quad \text{①}$$

$$\therefore t_c = \frac{45000 - 35000 - 0}{3758.32 \sqrt{\frac{1}{25} + \frac{1}{25}}} = 9.407 \quad \text{① point}$$

The critical value: $\alpha = .05 \Rightarrow t_{\alpha/2, n_1+n_2-2} = t_{0.025, 48} \approx 2.0086$ ①
with $df = 50$

Decision Rule:

Reject H_0 if $|t_c| > t_{\alpha/2, n_1+n_2-2}$

$$\Rightarrow 9.407 > 2.0086 \Rightarrow \text{Reject } H_0$$

① point

Conclusion:

The data do not provide a sufficient evidence to conclude that there is no difference between the average annual salaries of the two degrees. ① point

4. Figure perfect incorporation is a women's figure salon that specializes in weight reduction programs. Weights of a sample of 6 clients before and after a 6-week introductory program are shown below

| Weight | Before | 140 | 160 | 210 | 148 | 190 | 170 |
|--------|--------|-----|-----|-----|-----|-----|-----|
| | After | 132 | 158 | 195 | 152 | 180 | 164 |
| | D_i | 8 | 2 | 15 | -4 | 10 | 6 |

Test to determine whether the introductory program provides a statistically significant weight loss at 1% significance level.

The hypotheses are: $H_0: \mu_d = \mu_1 - \mu_2 \leq 0$ $H_A: \mu_d = \mu_1 - \mu_2 > 0$ ① point

The assumptions are: a. Samples are dependent b. Small sample - pairs size ① points

The test statistic value: μ_1 : Before μ_2 : After

$$t_c = \frac{\bar{d} - d_0}{s_d / \sqrt{n}} \quad , \quad \bar{d} = \frac{\sum d_i}{n} = \frac{37}{6} = 6.167 \quad \} \text{ ① point}$$

$$s_d = \sqrt{\frac{\sum (d_i - \bar{d})^2}{n-1}} = 6.5853 \quad \} \text{ ① point}$$

$$t_c = \frac{6.167 - 0}{6.5853 / \sqrt{6}} = 2.293 \quad \} \text{ ①}$$

The critical value: $\alpha = .01 \Rightarrow t_{\alpha, n-1} = t_{.01, 5} = 3.3649$ ① point

The Decision Rule: Reject H_0 if $t_c > t_{\alpha, n-1}$
 $\Rightarrow 2.293 \not> 3.3649 \quad \therefore$ Do not reject H_0 } ①

The Conclusion:

Based on the sample data, the introductory program does not provide a statistically significant weight loss. ① point

5. Consider question 3 above, do you think that the standard deviations of the annual salaries of both the Bachelor's degree and the Master's degree should be equal at 10% significance level?

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| The hypotheses are: $H_0: \sigma_1^2 = \sigma_2^2$ $H_A: \sigma_1^2 \neq \sigma_2^2$ ① point |
| The assumption is: 1. populations are normally distributed 2. Sample variances are indep. ② points |
| The test statistic value: $F_c = \frac{S_1^2}{S_2^2} = \frac{(4000)^2}{(3500)^2} = 1.3061$ } ② points |
| The critical value: $F_{\alpha/2, n_1-1, n_2-1} = F_{0.05, 24, 24} = 1.984$ ① point |
| The Decision Rule: Reject H_0 if $F_c > F_{\alpha/2, n_1-1, n_2-1}$ $\Rightarrow 1.3061 < 1.984 \Rightarrow$ Do not reject H_0 . ④ points |
| The Conclusion: Based on the sample data, the standard deviations of the annual salaries of both degrees are equal. ① point |

6. The filling variance for boxes of cereal is designed to be no more than 0.02. A sample of ³¹41 boxes of cereal shows a standard deviation of 0.16 ounces. Determine whether the filling specifications were violated. Use $\alpha = 0.05$

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|---|---|---------------------|
| The hypotheses are: $H_0: \sigma^2 \leq 0.02$ | $H_A: \sigma^2 > 0.02$ | ① point |
| The assumption is: Population is normally distributed. ① point | | |
| The test statistic value: | | |
| $\chi_c^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{(31-1)(0.16)^2}{0.02} = 38.4$ | | } ② points |
| The critical value | $\chi_{\alpha, n-1}^2 = \chi_{0.05, 30}^2 = 43.7730$ | (→ df = 30) ① point |
| The Decision Rule: | Reject H_0 if $\chi_c^2 > \chi_{\alpha, n-1}^2$ $\Rightarrow 38.4 \not> 43.7730$ \therefore Do NOT Reject H_0 | } ② points |
| The Conclusion: | Based on the sample data, The filling variance for boxes is <u>NO</u> more than 0.02. ① point <u>NOT violated</u> | |

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