Chapter 8

1. Testing hypotheses about μ

- **a.** For large sample or known σ , z values are
- **b.** For small sample and unknown σ , t values are used.

The test statistics are

$$z = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}}$$
 or $t_{n-1} = \frac{\overline{x} - \mu}{s / \sqrt{n}}$ will be

compared with

- **a.** z_{α} or t_{α} for one tailed hypotheses.
- **b.** $z_{\alpha/2}$ or $t_{\alpha/2}$ for two tailed hypotheses.

2. Testing hypotheses about p

The test statistic

$$z = \frac{\overline{p} - p}{\sqrt{p(1-p)/n}}$$
 where $\overline{p} = \frac{x}{n}$

1. Testing hypotheses about the match paired

The ith paired difference is $d_i = x_{1i} - x_{2i}$

Test statistic is
$$t_{n-1} = \frac{\overline{d} - \mu_d}{s_d / \sqrt{n}}$$

where
$$\overline{d} = \frac{1}{n} \sum_{i=1}^{n} d_i$$
 and $s_d = \sqrt{\sum_{i=1}^{n} (d_i - \overline{d})^2 / (n-1)}$.

2. Testing difference between two independent population means

a. If σ_1 and σ_2 known or n_1 and $n_2 \ge 30$

The test statistics is

$$z = \frac{\left(\overline{x}_{1} - \overline{x}_{2}\right) - \left(\mu_{1} - \mu_{2}\right)}{\sqrt{\left(\sigma_{1}^{2} / n_{1}\right) + \left(\sigma_{2}^{2} / n_{2}\right)}}$$

b. If σ_1 and σ_2 unknown or n_1 and $n_2 \ge 30$, the test

$$z = \frac{\left(\overline{x}_{1} - \overline{x}_{2}\right) - \left(\mu_{1} - \mu_{2}\right)}{\sqrt{\left(s_{1}^{2} / n_{1}\right) + \left(s_{2}^{2} / n_{2}\right)}}$$

c. If σ_1 and σ_2 unknown or n_1 or $n_2 < 30$ (assuming equal σ 's)

$$t_{n_1+n_2-2} = \frac{\left(\overline{x}_1 - \overline{x}_2\right) - \left(\mu_1 - \mu_2\right)}{s_p \sqrt{(1/n_1) + (1/n_2)}}$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

d. If σ_1 and σ_2 unknown or n_1 or $n_2 < 30$

$$t = \frac{\left(\overline{x}_{1} - \overline{x}_{2}\right) - \left(\mu_{1} - \mu_{2}\right)}{\sqrt{\left(s_{1}^{2} / n_{1}\right) + \left(s_{2}^{2} / n_{2}\right)}}$$

$$df = \frac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{\left(s_1^2/n_1\right)^2/(n_1 - 1) + \left(s_2^2/n_2\right)^2/(n_2 - 1)}$$

3. To test hypotheses about difference between two independent proportions

The test statistic is

$$z = \frac{\left(\overline{p}_{1} - \overline{p}_{2}\right) - \left(p_{1} - p_{2}\right)}{\sqrt{\overline{p} (1 - \overline{p})} \left((1/n_{1}) + 1/n_{2}\right)}$$

where
$$\overline{p} = \frac{n_1 \overline{p}_1 + n_2 \overline{p}_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$$
Chapter 10

1. Testing hypotheses about σ

The test statistic is

$$\chi^2 = (n-1)s^2 / \sigma_0^2$$

which has a χ^2 distribution with df = n - 1 and σ_0^2 is hypothetical.

2. Testing hypotheses about $\sigma_1^2 - \sigma_2^2$

From test statistic is
$$F_0 = s_1^2 / s_2^2 \text{ with } df_1 = n_1 - 1 \text{ and } df_2 = n_2 - 1.$$
Chapter 12

1. For Goodness-of-fit test, the statistic is

$$\chi^2 = \sum \frac{(o_i - e_i)^2}{e_i}$$
 with k – 1 degrees of freedom

 o_i = Observed cell frequency

 e_i = Expected cell frequency

2. For Test of Independence, the test statistic is

$$\chi^2 = \sum_{i=1}^r \sum_{j=1}^c \frac{(o_{ij} - e_{ij})^2}{e_{ij}}$$
 where

 $e_{ij} = (i^{th} row total)(j^{th} column total) / (sample size)$

Chapter 13

1. Sample correlation coefficient

$$r = \frac{\sum (x - \overline{x})(y - \overline{y})}{\sqrt{\sum (x - \overline{x})^2 \sum (y - \overline{y})^2}}$$
$$= \frac{n \sum xy - (\sum x)(\sum y)}{\sqrt{[n \sum x^2 - (\sum x)^2][n \sum y^2 - (\sum y)^2]}}$$

2. For testing hypotheses about ρ , the test statistic

$$t_{n-2} = r/\sqrt{(1-r^2)/(n-2)}$$
 has df=n-2

3. Estimated regression model

$$\hat{y}_i = b_0 + b_1 x$$

4. The Least Square Estimates are

$$b_{1} = \frac{\sum (x - \overline{x})(y - \overline{y})}{\sum (x - \overline{x})^{2}} = \frac{\sum xy - (\sum x \sum y)/n}{\sum x^{2} - (\sum x)^{2}/n}$$

$$b_0 = \overline{y} - b_1 \overline{x}$$

5. Total Sum of Squares

$$SST = \sum (y - \overline{y})^2 = \sum_{i=1}^{n} y_i^2 - n\overline{y}^2$$

6. Regression & Error Sum of Squares

$$SSR = \sum (\hat{y} - \overline{y})^2 = b_1 \left(\sum xy - \left(\sum x \sum y \right) / n \right)$$

$$SSE = \sum (y - \hat{y})^2 = SST - SSR$$

7. Coefficient of Determination

R- Squared =
$$R^2 = \frac{SSR}{SST}$$

$$R^2 = \frac{SST - SSE}{SST} = 1 - \frac{SSE}{SST} = r^2$$

8. Standard Error of the Estimate

$$s_{\varepsilon} = \sqrt{SSE/(n-k-1)}$$

9. Standard Deviation of the Slope

$$S_{b_1} = \frac{S_{\varepsilon}}{\sqrt{\sum (x - \overline{x})^2}} = \frac{S_{\varepsilon}}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{n}}}$$

For testing H_0 : $\beta_1 = 0$ vs. H_1 : $\beta_1 \neq 0$

10. The test statistic & C.I. for the slope

$$t_{n-2} = \frac{b_1 - \beta_1}{s_{b_1}} \& b_1 \pm t_{\alpha/2} s_{b_1}$$

11. C.I. for the mean of y given a particular x_p

$$\hat{y} \pm t_{\alpha/2} S_{\varepsilon} \sqrt{\frac{1}{n} + \left[\left(x_{p} - \overline{x} \right)^{2} / \sum (x - \overline{x})^{2} \right]}$$

12. C.I. estimate for an Individual value of y given a particular x_p

$$\hat{y} \pm t_{\alpha/2} s_{e} \sqrt{1 + \frac{1}{n} + \left[(x_{p} - \overline{x})^{2} / \sum (x - \overline{x})^{2} \right]}$$

Chapter 14

1. Estimated multiple regression model

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_k x_k$$

2. Two variable model is

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 \& e_i = y_i - \hat{y}_i$$

is Errors (residuals) from regression model

3. Proportion of variation in y explained by xadjusted for the number of x variables used

$$R_A^2 = 1 - \left(1 - R^2\right) \left(\frac{n-1}{n-k-1}\right)$$

4. For testing
$$\beta_1 = \beta_2 = ... = \beta_k$$
 Test statistic
$$F = \frac{SSR / k}{SSE / (n - k - 1)} = \frac{MSR}{MSE}$$

with $df_1 = k$ and $df_2 = n - k - 1$

5. For testing H_0 : $\beta_i = 0$ vs. H_A : $\beta_i \neq 0$

The test statistic & C.I. for the slope β_i are

$$t_{n-k-1} = \frac{b_i - 0}{s_{b_i}}$$
 & $b_i \pm t_{\alpha/2} s_{b_i}$, respectively.

5. The estimate of the standard deviation of the regression model is

$$s_{\varepsilon} = \sqrt{SSE / (n - k - 1)} = \sqrt{MSE}$$
 & $VIF_{j} = \frac{1}{1 - R_{j}^{2}}$

is the Variance Inflationary Factor (VIFj)

Chapter 15

1. Simple Index number formula & Unweighted aggregate price index formula (respectively)

$$I_t = \frac{y_t}{y_0} 100 \& I_t = \frac{\sum p_t}{\sum p_0} 100$$

2. Paasche & Laspeyres Weighted Aggregate **Price Indexes (respectively)**

$$I_{t} = \frac{\sum q_{t} p_{t}}{\sum q_{t} p_{0}} 100 \& I_{t} = \frac{\sum q_{0} p_{t}}{\sum q_{0} p_{0}} 100$$

3. Deflation formula

$$y_{adj_t} = \frac{y_t}{I_t} 100$$

4. Forecasting formula & Residual formula are

$$F_t = \hat{y} = b_0 + b_1 t$$
 & $e_t = y_t - F_t$ respectively.

5. Mean Square Error & Mean Absolute **Deviation are (respectively)**

$$MSE = \sum (y_t - F_t)^2 / n \& MAD = \sum |y_t - F_t| / n$$

6. For testing H_0 : $\rho=0$ vs. H_A : $\rho\neq0$ **Durbin-Watson Test statistic**

$$d = \sum_{i=1}^{n} (e_{t} - e_{t-1})^{2} / \sum_{i=1}^{n} e_{t}^{2}$$

7. Multiplicative Time-Series Model

$$y_t = T_t \times S_t \times C_t \times I_t$$

 T_t = Trend value S_t = Seasonal value

 C_t = Cyclical value I_t = Irregular (random) value

8. Ratio-to-Moving Average formula & Deseasonalizing formula (respectively)

Deseasonanzing formula (respective)

$$S_t \times I_t = \frac{y_t}{T_t \times C_t} \& T_t \times C_t \times I_t = \frac{y_t}{S_t}$$

9. Single Exponential Smoothing Model

$$F_{t+1} = F_t + \alpha(y_t - F_t) = \alpha y_t + (1 - \alpha)F_t$$
 where

α: smoothing constant.

10. Double Exponential Smoothing Model

$$C_{t} = \alpha y_{t} + (1 - \alpha)(C_{t-1} + T_{t-1})$$

$$T_t = \beta (C_t - C_{t-1}) + (1 - \beta)T_{t-1} \& F_{t+1} = C_t + T_t$$

- α : Constant-process smoothing constant
- β: Trend-smoothing constant