Question.1. (6-Points) * Solutions*

Answer True or False

1. If you are about to buy a new car , suppose that the probability that you will buy a Japanese car (E_1) is 0.85, a German car (E_2) is 0.75 then the two events E_1 and E_2 are independent : ----False

T 2. The number of customers who entered " Al-Rashid Mall" between 4:00 pm and 6:00pm yesterday is an example of discrete random variable:---True

F 3. The hypergeometric probability distribution is used rather than the binomial or the Poisson when the sampling is performed without replacement from an infinite population:--False

T 4. A discrete random variable may take an infinite countable set of numbers:--True

T 5. A normal distribution is defined by knowing the mean and the standard deviation.--True

F 6. The standard deviation for exponential distribution exceeds the mean.-False

Question.2. (6-Points)

Answer the following questions by choosing the right answer.

1. If the probability that Mr. Ahmed will get a scholarship for higher studies if he becomes the first honor is higher than it if he becomes the second one, then the two events A: Getting a scholarship for higher studies and B: Being the first honor are:

   a. Dependent.
   b. Independent.
   c. Mutually exclusive.
   d. None of the above

2. Which one of the following statements is **FALSE**?

   a. Any event from the sample space and its complement are mutually exclusive.
   b. A random variable may be discrete or continuous.
   c. The expected value of both the discrete and continuous random variables must be always one.
   d. The conditional probability of two mutually exclusive events always zero

Solutions

3. If a study is set up in such a way that a sample of people is surveyed to determine whether they have ever used a particular product, the likely probability distribution that would describe the random variable—the number who say yes—is a:

a. Binomial distribution.

b. Poisson distribution.

c. Uniform distribution.

d. Normal distribution

4. Which of the following is not a condition of the binomial distribution?

a. Two possible outcomes for each trial

b. The trials are independent

c. The standard deviation is equal to the square root of the mean

d. The probability of a success remains constant from trial to trial

5. Which of the following probability distributions would most likely be used to describe the time between failures for electronic components?

a. Binomial distribution

b. Exponential distribution

c. Uniform distribution

d. Normal distribution

6. It is assumed that the time customers spend in a record store is uniformly distributed between 3 and 13 minutes. What is the probability that a customer will spend exactly 9 minutes in the record store?

a. 0.33

b. 0

c. 0.67

d. 0.4

Question3. (2+4+3+4 = 13 Points) *Solutions*

A total of 500 married working couples were polled about whether their annual salaries exceeded $25,000. The following information was obtained:

Wife	Husband: Less than $25,000	Husband: More than $25,000	Total
Less than $25,000	212	198	410
More than $25,000	36	54	90
Total	248	252	500

One of the couples is randomly chosen

a. What is the probability that the husband earns less than $25,000?

$$p(\text{The husband earns less than } \$25{,}000) = \frac{248}{500} = 0.496 \quad \text{(2) pts}$$

b. If the husband earns more than $25,000 what is the probability that his wife earns more than this amount?

Let E_1: The husband earns more than $25,000

E_2: His wife earns more than $25,000

$$p(E_2 \mid E_1) = \frac{p(E_1 \cap E_2)}{p(E_1)} = \frac{54/500}{252/500} = \frac{54}{252} = \frac{3}{14} = 0.2143$$

(1) (1) (1) (1)

c. What is the probability that the husband or his wife earn less than $25,000?

Let F_1: The husband earns Less than $25000

F_2: His wife " " " "

$$p(F_1 \cup F_2) = p(F_1) + p(F_2) - p(F_1 \cap F_2) = \frac{248}{500} + \frac{410}{500} - \frac{212}{500} = \frac{223}{250} = 0.892. \quad (1)$$

(1) (1)

d. Are the two events: A: a husband earns more than $25,000, B: His wife earns less than $25,000 independent? Explain.

$$p(A) = \frac{252}{500} = 0.504, \quad p(B) = \frac{410}{500} = 0.82 \quad (1)$$

$$p(A \cap B) = \frac{198}{500} = 0.396 \quad (1)$$

$$p(A) \cdot p(B) = (.504)(.82) = 0.41328 \neq p(A \cap B) = 0.396 \quad (1)$$

$\therefore$ A & B are not independent. (1)

Solutions

Question4. (2+4+2+4 = 12-Points)

The number of overnight emergency calls (X) to the answering service of a heating and air conditioning firm and their probabilities are summarized in the following table

X	0	1	2	3	4	5
P(x)	0.05	0.10	0.15	0.35	0.20	0.15

a. What is the probability that the answering service will receive at most 3 calls?

$$p(X \le 3) = p(0) + p(1) + p(2) + p(3) = 0.05 + 0.10 + 0.15 + 0.35 \Big\} \; 1$$

$$= 0.65 \; \} \; 1$$

b. Find the probability that the answering service will receive at least 2 calls given that it received more than 3 calls.

Let A : The answering service will receive at least 2-Calls

$A = \{2, 3, 4, 5\} \rightarrow 1$

B : The answering service received more than 3 calls

$1 \leftarrow B = \{4, 5\} \Rightarrow A \cap B = \{4, 5\} \rightarrow 1$

$$p(A \mid B) = \frac{p(A \cap B)}{p(B)} = \frac{p(4) + p(5)}{p(4) + p(5)} = 1 \quad 1$$

c. Find the expected value for the number of overnight emergency calls

$$E(X) = \sum x\, p(x)$$

$$= (0)(.05) + (1)(0.10) + (2)(.15) + (3)(.35) + (4)(.20) + (5)(.15)$$

$$2 \quad = 0 + .10 + 0.30 + 1.05 + .80 + .75$$

$$= 3.0$$

d. Find the standard deviation for the number of overnight emergency calls.

$$\sigma = \sqrt{\Sigma (x - \mu)^2\, p(x)}$$

$$= \sqrt{(0-3)^2(.05) + (1-3)^2(.1) + (2-3)^2(.15) + (3-3)^2(.35) + (4-3)^2(.2) + (5-3)^2(.15)}$$

$$= \sqrt{0.45 + 0.40 + 0.15 + 0 + 0.20 + 0.60} \quad 3$$

$$= \sqrt{1.8}$$

$$= 1.3416 \; \} \; 1$$

Question5. (3+3+3 =9-Points) *Solutions*

Suppose that each customer who enters an electronic store will buy a television with probability 0.3, a radio with probability 0.15 and will not buy any thing with probability 0.55. For the next five customers (i.e., n = 5)

a. What is the probability that three customers exactly will buy three televisions?

$n=5, p=.3 \Rightarrow q = 1-.3 = .7$ } 1

$p(x=3) = C^5_3 \cdot (.3)^3(.7)^2$
$= (10)(0.027)(.49)$ } 1
$= 0.1323$ } 1

b. What is the probability that at least one customer will buy a radio?

$n=5, p=.15 \Rightarrow q = 1-.15 = .85$ } 1

$p(x \geq 1) = 1 - p(x<1) = 1 - p(0)$
$= 1 - C^5_0 \cdot (.15)^0 (.85)^5$ } 1
$= 1 - 0.4437$
$= 0.5563$ } 1

c. Find the expected number of customers and the standard deviation who will not buy any thing?

$n=5, p=.55 \Rightarrow q = 1-.55 = .45$ 1

I. $E(X) = np = (5)(.55) = 2.75$ } 1

II. $\sigma = \sqrt{npq} = \sqrt{(5)(.55)(.45)} = \sqrt{1.2375}$
$= 1.1124$ } 1

Question6. (2+3 = 5-Points)

If cars arrive at an automobile service center randomly and independently at a rate of 5 per hour on average, then:

a. What is the probability of the service center being totally empty in a given hour?

$\lambda = 5, t = 1 \Rightarrow \lambda t = 5$

$p(x=0) = \frac{5^0 e^{-5}}{0!} = e^{-5}$
$= 0.0067$ } 2

b. What is the probability that exactly 8 cars will be in the service center during a period of two hours?

$\lambda = 5, t = 2 \rightarrow \lambda t = (5)(2) = 10$ 1

$p(x=8) = \frac{(10)^8 e^{-10}}{8!}$
$= 0.1126$ } 2

Question 7 (4+4+4 = 12-Points) * Solutions *

The manager of a small postal substation is trying to quantify the variation in the weekly demand for mailing tubes. The manager decided to assume that this demand is normally distributed with an **average of 100** tubes purchased weekly. And he found that 90 percent of the time, the weekly demand is below 115, then:

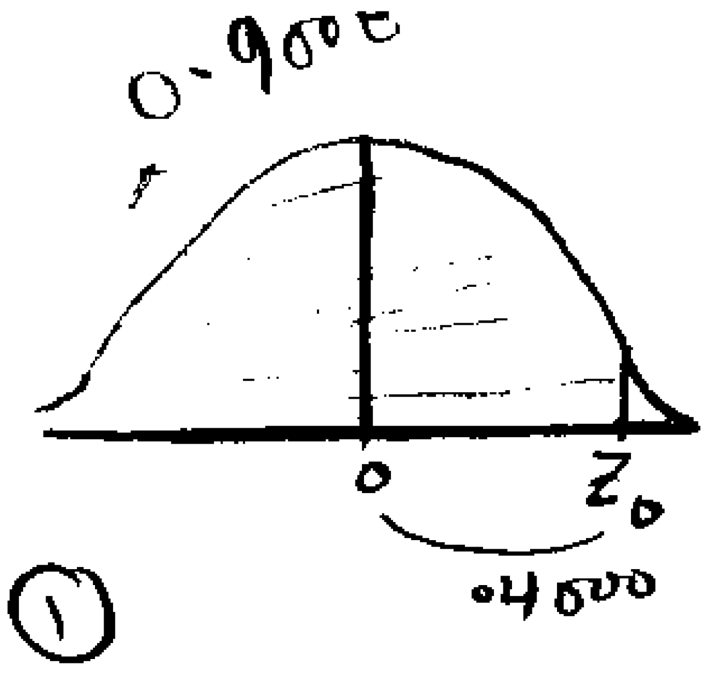
a. Find the **standard deviation** of this distribution.(Round your answer in this part to 2 decimal places)

$X \sim N(\mu = 100, \sigma = ?)$

$p(X \le 115) = 0.9000$

$p\left(\frac{X-100}{\sigma} \le \frac{115-100}{\sigma}\right) = .9000 \to 1$

$= p\left(Z \le \frac{15}{\sigma}\right) = .9000$, let $Z_0 = \frac{15}{\sigma}$



$\Rightarrow p(0 \le Z \le Z_0) = 0.9000 - .5000 = .4000 \} 1$

$\Rightarrow Z_0 = 1.28 \to 1$

$\therefore \frac{15}{\sigma} = \frac{1.28}{1} \Rightarrow 1.28\sigma = 15 \Rightarrow \sigma = \frac{15}{1.28} = 11.72$ 1

b. What is the probability that between 99 and 113 tubes will purchased in a certain week?

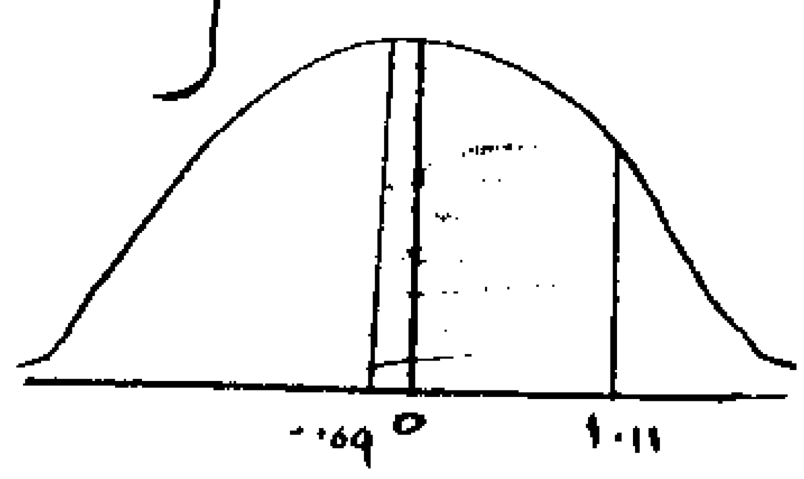
$p(99 \le X \le 113) = p\left(\frac{99-100}{11.72} < \frac{X-100}{11.72} \le \frac{113-100}{11.72}\right) \} 2$

$= p(-0.09 \le Z \le 1.11)$

$= p(0 \le Z \le .09) + p(0 \le Z \le 1.11)$

$= 0.0359 + 0.3665 \} 1$

$= 0.4024 \} 1$

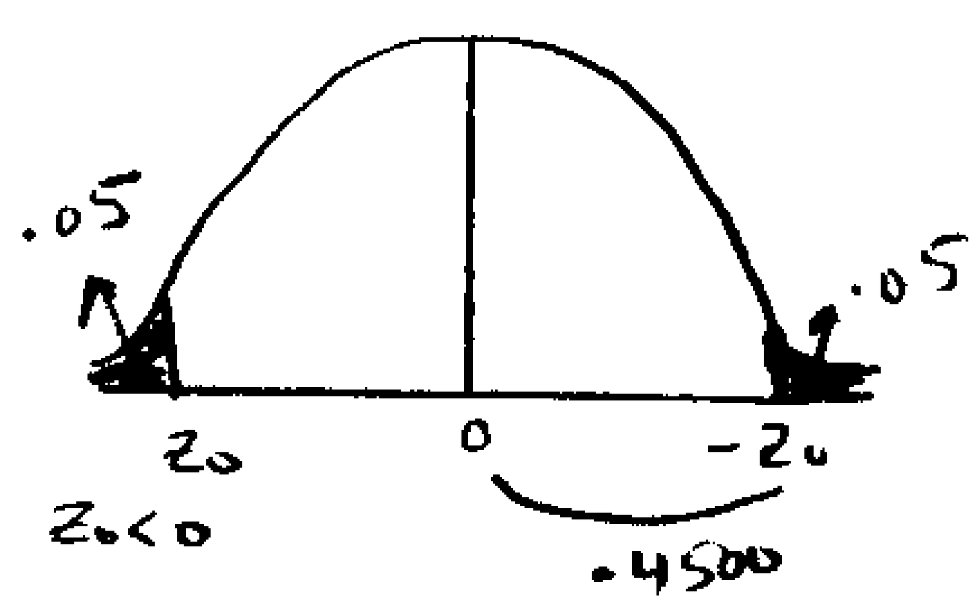


c. The manager wants to stock enough mailing tubes each week so that the probability of running out of the tubes is no higher than 0.05.Find the lowest number of such stocks

Let the lowest number of stocks be k

$p(X < k) = .05$ 1

$p\left(\frac{X-100}{11.72} \le \frac{k-100}{11.72}\right) = .05$, let $Z_0 = \frac{k-100}{11.72}$



1 $p(Z < Z_0) = p(Z > -Z_0) = .05$

$\Rightarrow p(0 \le Z < -Z_0) = .5 - .05 = .45 \Rightarrow -Z_0 = 1.645 \Rightarrow Z_0 = -1.645$ 1

$\therefore \frac{k-100}{11.72} = \frac{-1.645}{1} \Rightarrow k - 100 = -19.2794$

$k = -19.2794 + 100 = 80.7206 \simeq 81$ tubes

1

Question8. (2+3+2 = 7-Points) * Solutions *

A study of cars arriving at a parking structure at the local airport shows that the time between arrivals is 1.2 minutes and is exponentially distributed.

a. Find the probability that more than two minutes will elapse between the arrivals of cars. $\lambda = 1.2$ minutes

$$\left.\begin{aligned} p(X>2) &= 1 - p(X \le 2) \\ &= 1 - (1 - e^{-(1.2)(2)}) \\ &= 1 - 1 + e^{-2.4} \\ &= e^{-2.4} \\ &= 0.0907 \end{aligned}\right\} \quad 2$$

b. How many minutes will elapse between the arrivals of 90 percent of the cars?(Hint: Find the 90[th] percentile) Let k = 90-th percentile

$p(X \le k) = .90$

$$\left. 1 - e^{-\lambda k} = .90 \Rightarrow 1 - e^{-1.2k} = .9 \right\} \quad 1$$

$$e^{-1.2k} = .1$$

$$-1.2k = \ln(.1) \Rightarrow k = \frac{\ln(.1)}{-1.2} = 1.92 \text{ minutes} \quad 1$$

1

c. Find the mean number of cars arriving per 1.2 hours.

It is a Poisson dist. with mean $= \frac{1}{1.2}$ Cars per minute 1

$$\left.\begin{aligned} \text{The mean per 1.2 hours} &= \frac{1}{1.2}(1.2 * 60) \\ &= 60 \text{ Cars per 1.2 hrs.} \end{aligned}\right\} \quad 1$$