Question .1(*4+6=10-Points*) Solutions − B − 1

Ahmed followed stock exchanges over the past 50 days. In particular, he recorded price exchanges for two stocks, Al- Kahraba' and Safco. He partially constructed the following table.

		Safco price↓			Total
		Decrease	Unchanged	Increase	
Al-Kahraba ' price →	Decrease	50	70	70	190
	Unchanged	10	50	40	100
	Increase	60	80	70	210
Total		120	200	180	500

With this method, he partially completed the following table to study the behavior of the two stocks.

		Safco↓			Total
		Decrease	Unchanged	Increase	
Al-Kahraba ' →	Decrease	0.10	0.14	0.14	038
	Unchanged	0.02	0.10	0.08	0.20
	Increase	0.12	0.16	0.14	0.42
Total		0.24	0.40	0.36	1

(1)

With this partially constructed table, find the following:

a. What is the probability that Safco' stock price decreases given that Al-Kahraba' price decreases?

Let E_1: Safco' stock price decreases
E_2: AL-Kahraba' stock price decreases

$$p(E_1 | E_2) = \frac{p(E_1 \cap E_2)}{p(E_2)} = \frac{0.10}{0.38} = 0.2632$$

(1) (1) (1)

b. Let A: Safco decreases and B: Al-Kahraba' stock decreases.

I. Are these two events mutually exclusive? Why?

NO, because : $p(A \cap B) = 0.10 \neq 0$

(1) (1)

II. Are these two events independent? Why?

(1) NO, because : $p(A \cap B) = .10 \neq p(A) \cdot p(B) = (.24)(.38) = 0.0912$

(1)

III. Find $P(A \cup B)$

$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$
$$= .24 + .38 - .10 \quad (1)$$
$$= 0.52 \quad (1)$$

Question .2 (*1+2+4+2+4=13-Points*) * Solutions - B - 2

The following distribution of number of daily customer complaints was observed for the past year at Giant Supermarket

X	0	1	2	3	4	5
P(X)	0.15	0.30	0.20	0.15	0.14	0.06

a. What type of probability distribution is represented above?

Discrete prob. distribution } ①

b. Find the probability that on a given day, there will be at most one customer complaint?

$$p(\text{At most one}) = p(x \le 1) = p(0) + p(1) \quad \} ①$$
$$= .15 + .30 = 0.45 \quad \} ①$$

c. Find the probability that on a given day, there will be between 2 and 4 complaints (inclusive) given that there is at most three complaint.

Let A: There will be between 2 & 4 complaints $\Rightarrow 2 \le x \le 4 \rightarrow \{2,3,4\}$

B: " is at most 3 complaints $\Rightarrow x \le 3 \rightarrow \{0,1,2,3\}$

① $A \cap B = \{2,3\}$ ①

$$p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{p(2)+p(3)}{p(0)+p(1)+p(2)+p(3)} = \frac{.2+.15}{.15+.30+.20+.15}$$
①
$$= \frac{.35}{0.80} = 0.4375 \quad ①$$

d. Find the expected number of customer complaints?

$$E(x) = \Sigma x\, p(x) = (0)(.15)+(1)(.30)+(2)(.20)+(3)(.15)+(4)(.14)+(5)(.06) \quad \} ①$$
$$= 0 + .30 + .40 + .45 + .56 + .30$$
$$= 2.01 \quad \} ①$$

e. Find the standard deviation of customer complaints?

$$\sigma = \sqrt{\Sigma(x - E(x))^2 p(x)}$$
$$= \sqrt{(0-2.01)^2(.15) + (1-2.01)^2(.3) + (2-2.01)^2(.2) + (3-2.01)^2(.15) + (4-2.01)^2(.14) + (5-2.01)^2(.06)} \quad \} ②$$
$$= \sqrt{0.606015 + 0.30603 + 0.00002 + .147015 + 0.554414 + 0.536406}$$
$$= \sqrt{2.1499} = 1.4663.$$
① ①

Question .3(3+4+3=10-Points) * Solutions - B - 3

The life time of batteries manufactured by a factory has an exponential distribution with mean 360 hours. A battery is selected randomly from the product of the factory. Then:

a. Find the probability that the battery will work at most 320 hours.

The mean $= 360 = \frac{1}{\lambda} \Rightarrow 360\lambda = 1 \Rightarrow \lambda = \frac{1}{360}$ (1)

$p(\text{At most } 320) = p(0 \le X \le 320)$

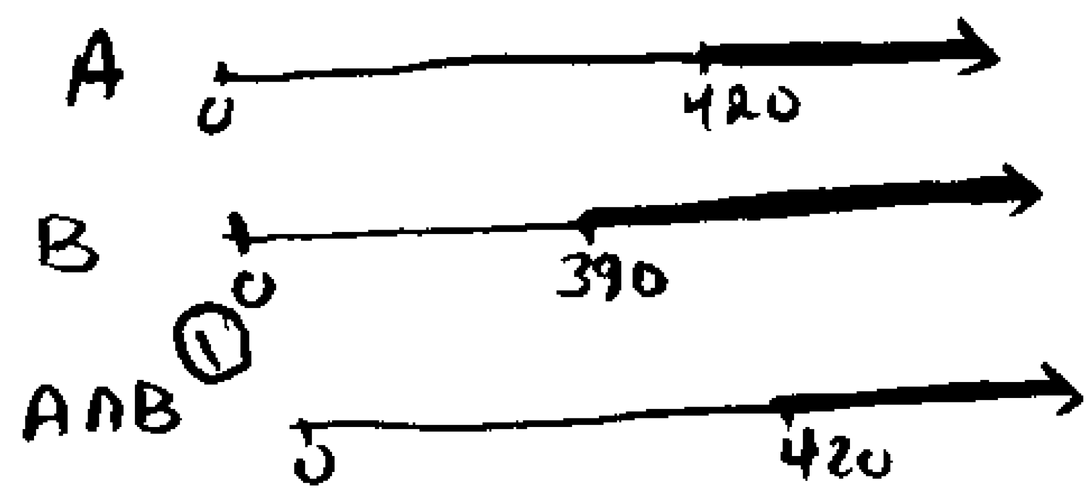
$= 1 - e^{-\frac{1}{360}\cdot 320} = 1 - e^{-0.8889}$ (1)

$= 1 - 0.4111 = 0.5889.$ (1)

b. Find the probability that the battery will work more than 420 hours given that it has worked more than 390 hours.

Let A : The battery will work more than 420 hours $\Rightarrow X > 420$

B : " " worked more than 390 $\Rightarrow X > 390$



$p(A|B) = \frac{p(A\cap B)}{p(B)} = \frac{p(X > 420)}{p(X > 390)}$ (1)

$= \frac{1 - p(0 \le X \le 420)}{1 - p(0 \le X \le 390)} = \frac{1 - (1 - e^{-420/360})}{1 - (1 - e^{-390/360})}$ (1)

$= \frac{e^{-1.1667}}{e^{-1.0833}} = \frac{0.3114}{0.3385} = 0.9199.$ (1)

c. Find the median of the life time of the battery.

The median $= 50^{th}$ percentile $= P_{50}$

$p(X \le P_{50}) = .5$ } (1)

$\Rightarrow p(0 \le X \le P_{50}) = .5$

$1 - e^{-\frac{P_{50}}{360}} = .5$ } (1)

$e^{-\frac{P_{50}}{360}} = .5$ Take ln for both sides

$-\frac{P_{50}}{360} = -0.6931$

$\Rightarrow P_{50} = (360)(0.6931) = 249.516$ hours. } (1)

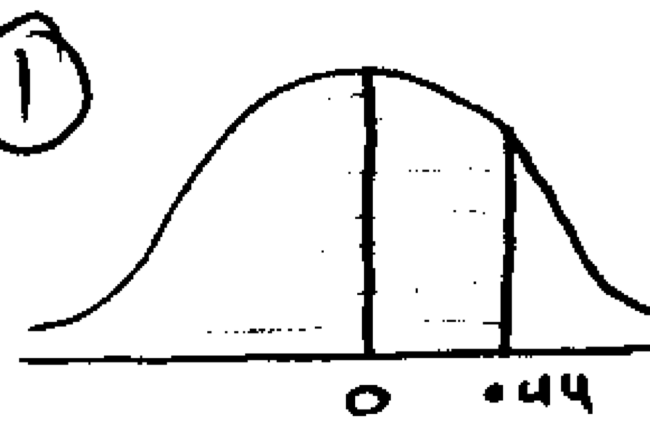
Question .4(3+4+4+4=15-Points) * Solutions - B - 4

At KFUPM the distribution of student after-class daily studying time has been known to follow a ***normal distribution*** with a ***mean*** of ***110*** minutes and a ***standard deviation*** of ***36*** minutes.

a. A KFUPM student is randomly selected, what is the probability that he studies less than 126 minutes?

$$X \sim N(\mu = 110,\ \sigma = 36)$$

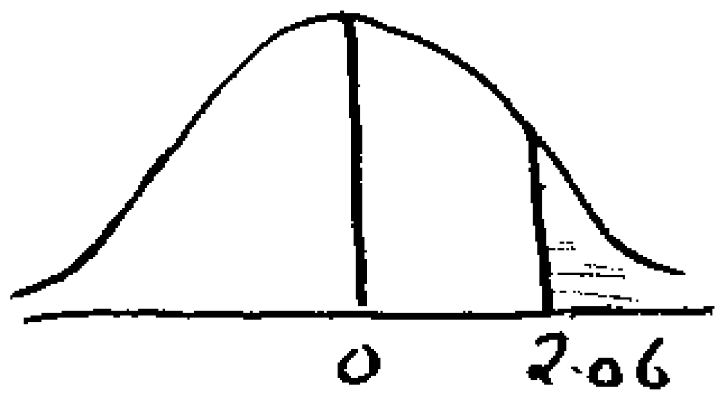
$$p(X < 126) = p\left(\frac{X-110}{36} < \frac{126-110}{36}\right) \quad 1$$



$$= p(Z < 0.44)$$
$$= 0.5000 + p(0 \le Z \le .44)$$
$$= .5000 + .1700 \rightarrow 1$$
$$= 0.6700 \quad 1$$

b. If students who typically obtain A+ grades in their courses study at least 184 minutes [redacted] daily, what is the percentage of these KFUPM students?

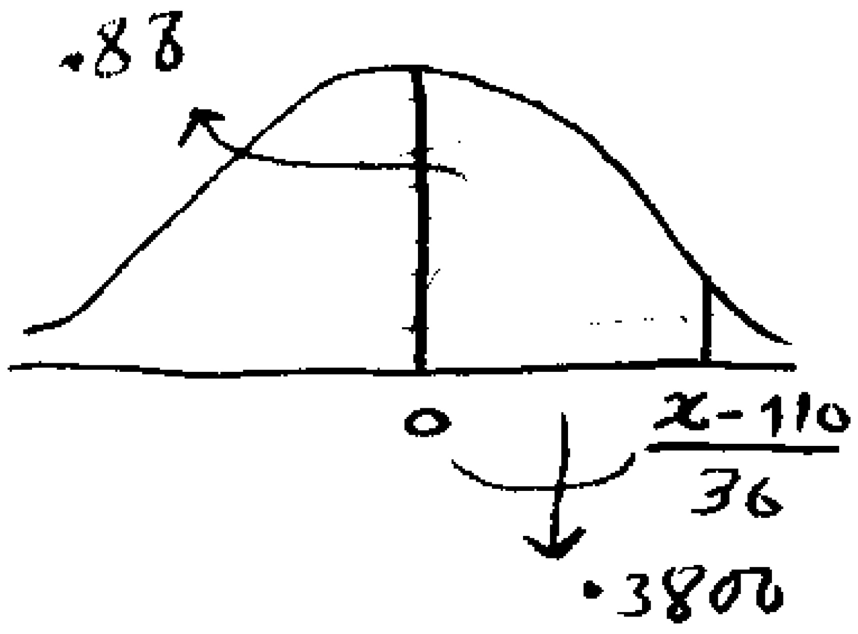
$$p(X > 184) = p\left(\frac{X-110}{36} > \frac{184-110}{36}\right) \Big\} 1$$



$$= p(Z > 2.06)$$
$$= 0.5 - p(0 \le Z < 2.06) \quad 1$$
$$\left.\begin{array}{l} = 0.5 - .4803 = 0.0197 \\ \text{or, their percentage} = (0.0197)(100\%) = 1.97\% \end{array}\right\} 2$$

c. Find x where 88% of the students study less than x minutes [redacted]

$$p(X \le x) = .8800 \Big\} 1$$
$$p\left(\frac{X-110}{36} \le \frac{x-110}{36}\right) = .8800 \Big\} 1$$



$$p\left(Z < \frac{x-110}{36}\right) = .8800$$
$$\Rightarrow p\left(0 \le Z < \frac{x-110}{36}\right) = .8800 - .5 = 0.3800$$
$$\Rightarrow \underbrace{\frac{x-110}{36} = 1.175}_{1} \Rightarrow \left.\begin{array}{l} x = (36)(1.175) + 110 \\ = 152.3 \end{array}\right\} 1$$

d. If 8 KFUPM students are selected at random, then find the probability that at most one of them will study less than 126 minutes.

In this case we have a binomial dist. with $n = 8$ and 1

p: prob. of success $= p(X < 126) = .67$ (From part (a)) 1

$\Rightarrow q = 1 - .67 = .33$

$$\left.\begin{array}{l} p(\text{At most one}) = p(X \le 1) \\ = p(0) + p(1) \end{array}\right\} 1$$

$$\left.\begin{array}{l} = C^8_0 (.67)^0 (.33)^8 + C^8_1 . (.67)^1 (.33)^7 \\ = 0.00014064 + 0.002284348 \\ = 0.0024. \end{array}\right\} 1$$

Question .5 (*3+2=5-Points*) Solutions - B - 5

The percentage of students who will be admitted to the university after taking an entrance exam is 66%. A random sample of 9 students from those who took the entrance exam is selected. Then:

a. Find the probability that 5 from them will be admitted to the university.

$X \sim$ binomial with $n=9$, $p=.66$, $q=1-.66=.34$ ①

$p(X=5) = C^9_5 \cdot (.66)^5 (.34)^4$
$= 0.2109$ } ②

b. Find the expected number of students in the sample who will be admitted to the university.

$E(X) = np$
$= (9)(.66) = 5.94$ } ②

Question .6(*4+4+4=12-Points*)

Suppose that on the average there are 5 car accidents weekly at the 3rd street. Then:

a. Find the probability that there will be at most 1 car accidents at the 3rd street next week.

$\lambda = 5$, $t = 1$

$p(\text{At most } 1) = p(X \leq 1) = p(0) + p(1)$ } ①

$= \frac{(5)^0 e^{-5}}{0!} + \frac{(5)^1 e^{-5}}{1!}$ } ②

$= 0.0067 + 0.0337 = 0.0404$ } ①

b. Find the probability that there will be at least 2 car accident at the 3rd street in the coming 2 weeks.

$\lambda = 5$, $t = 2 \Rightarrow \lambda t = (5)(2) = 10$ } ①

$p(\text{At least } 2) = p(X \geq 2)$
$= 1 - p(X<2)$ } ①
$= 1 - (p(0)+p(1)) = 1 - \left(\frac{(10)^0 e^{-10}}{0!} + \frac{(10)^1 e^{-10}}{1!}\right)$ } ①
$= 1 - (.000045399 + .00045399) = 1 - .0005 = .9995$ } ①

c. Find the **mean** and **standard deviation** of the number of car accidents in two years. (Hint: Use one year =53 weeks)

$\lambda = 5$, $t = (53)(2) = 106$ weeks

The mean $= E(X) = \lambda t = (5)(106) = 530$ } ②

The standard deviation $= \sigma = \sqrt{\lambda t} = \sqrt{530} = 23.0217$ } ②