Question .1(4+6=10-Points) * Solutions – A-1

Ahmed followed stock exchanges over the past 50 days. In particular, he recorded price exchanges for two stocks, Al- Kahraba' and Safco. He partially constructed the following table.

		Safco price↓			Total
		Decrease	Unchanged	Increase	
Al-Kahraba ' price →	Decrease	50	10	60	120
	Unchanged	70	50	80	200
	Increase	70	40	70	180
Total		190	100	210	500

With this method, he partially completed the following table to study the behavior of the two stocks.

		Safco↓			Total
		Decrease	Unchanged	Increase	
Al-Kahraba ' →	Decrease	0.10	0.02	0.12	0.24
	Unchanged	0.14	0.10	0.16	0.40
	Increase	0.14	0.08	0.14	0.36
Total		0.38	0.20	0.42	1

(1)

With this partially constructed table, find the following:

a. What is the probability that Al-Kahraba' stock price decreases given that Safco price decreases?

Let. E: AL-Kahraba' stock price decreases

F: Safco stock price decreases

$$p(E|F) = \frac{P(E\cap F)}{p(F)} = \frac{0.10}{0.38} \begin{matrix}\rightarrow (1)\\ \rightarrow (1)\end{matrix}$$

$$= 0.2632 \rightarrow (1)$$

b. Let A: Safco increases and B: Al-Kahraba' stock increases.

I. Are these two events mutually exclusive? Why?

(1) No, because:

$p(A\cap B) = 0.14 \neq 0$ (1)

II. Are these two events independent? Why?

(1) No, because:

$p(A) = .42$, $p(B) = .36$ $\Rightarrow p(A)\cdot p(B) = (.42)(.36) = 0.1512$

$p(A)\cdot p(B) \neq p(A\cap B)$ (1)

III. Find $P(A \cup B)$

$$p(A\cup B) = p(A) + p(B) - p(A\cap B)$$

$$= .42 + .36 - .14 \quad (1)$$

$$= 0.64. \quad (1)$$

*Solutions - A-2

Question .2 ***(1+2+4+2+4=13-Points)***

The following distribution of number of daily customer complaints was observed for the past year at Giant Supermarket

X	0	1	2	3	4	5
P(X)	0.2	0.25	0.25	0.1	0.1	0.1

a. What type of probability distribution is represented above?

Discrete prob. distribution (1)

b. Find the probability that on a given day, there will be at least one customer complaint?

$$p(X \geq 1) = 1 - p(X < 1) = 1 - p(X=0) \quad \} (1)$$
$$= 1 - 0.2 = 0.8 \quad \} (1)$$

c. Find the probability that on a given day, there will be between 2 and 4 complaints (inclusive) given that there is at least one complaint.

A : There will be between 2 & 4 complaints $\Rightarrow 2 \leq X \leq 4 \rightarrow \{2,3,4\}$

B : " is at least one complaint. $\Rightarrow X \geq 1 \rightarrow \{1,2,3,4\}$

$A \cap B = \{2,3,4\}$ (1)

$$p(A|B) = \frac{P(A \cap B)}{p(B)} \quad (1) \quad (1)$$

$$= \frac{p(2)+p(3)+p(4)}{1-p(0)} = \frac{.25+.1+.1}{.8} = \frac{.45}{.8} = 0.5625 \quad (1)$$

d. Find the expected number of customer complaints?

$$E(X) = \sum X\, p(x)$$
$$= (0)(.2) + (1)(.25) + (2)(.25) + (3)(.1) + (4)(.1) + (5)(.1) \quad \} (1)$$
$$= 0 + .25 + .5 + .3 + .4 + .5$$
$$= 1.95 \quad \} (1)$$

e. Find the standard deviation of customer complaints?

$$\sigma = \sqrt{\sum (x - E(x))^2 p(x)}$$
$$= \sqrt{(0-1.95)^2(.2) + (1-1.95)^2(.25) + (2-1.95)^2(.25) + (3-1.95)^2(.1) + (4-1.95)^2(.1) + (5-1.95)^2(.1)} \quad \} (2)$$
$$= \sqrt{.7605 + .225625 + .000625 + .11025 + .42025 + .93025}$$
$$= \sqrt{2.4475} = 1.5644.$$
(1) (1)

Question .3(3+4+3=10-Points) Solutions – A-3

The life time of batteries manufactured by a factory has an exponential distribution with mean 320 hours. A battery is selected randomly from the product of the factory. Then:

a. Find the probability that the battery will work at most 300 hours.

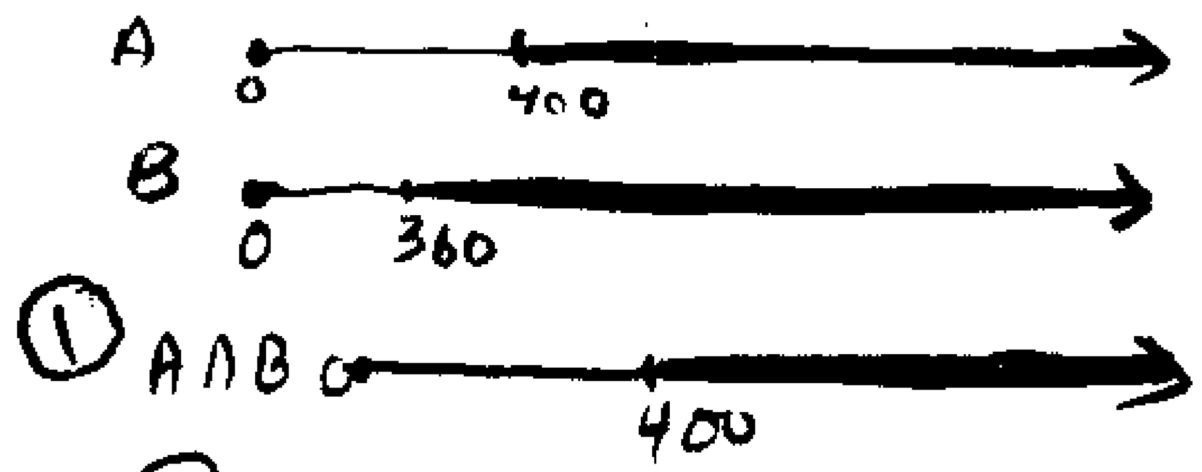
The mean $= \frac{320}{1} = \frac{1}{\lambda} \Rightarrow 320\lambda = 1 \Rightarrow \lambda = \frac{1}{320}$ 1

$$p(\text{At most } 300) = p(0 \le x \le 300)$$
$$= 1 - e^{-\frac{1}{320} \cdot 300} = 1 - e^{-0.9375} \quad 1$$
$$= 1 - 0.3916$$
$$= 0.6084. \quad 1$$

b. Find the probability that the battery will work more than 400 hours given that it has worked more than 360 hours.

Let A : The battery will work more than 400 hours $\rightarrow x > 400$

B : " " worked more than 360 hours $\rightarrow x \geq 360$



$$p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{p(x > 400)}{p(x > 360)}$$
1
$$= \frac{1 - p(0 \le x \le 400)}{1 - p(0 \le x \le 360)}$$
1
$$= \frac{1 - (1 - e^{-400/320})}{1 - (1 - e^{-360/320})} = \frac{e^{-1.25}}{e^{-1.125}} = \frac{0.2865}{0.3247} = 0.8824.$$
1

c. Find the median of the life time of the battery.

The median $= 50^{th}$ percentile $= P_{50}$

$$p(x \le P_{50}) = .5 \quad 1$$

$$\Rightarrow p(0 \le x \le P_{50}) = .5$$
$$\left.\begin{aligned} &1 - e^{-\frac{P_{50}}{320}} = .5 \\ &e^{-P_{50}/320} = .5 \end{aligned}\right\} 1$$
take (ln) for both sides:

$$-\frac{P_{50}}{320} = \ln .5 = -0.6931$$

$$\Rightarrow P_{50} = (320)(.6931) = 221.8071 \text{ hours.} \quad \} 1$$

Question .4(3+4+4+4=15-Points) Solutions – A-4

At KFUPM the distribution of student after-class daily studying time has been known to follow a ***normal distribution*** with a ***mean*** of ***100*** minutes and a ***standard deviation*** of ***30*** minutes.

a. A KFUPM student is randomly selected, what is the probability that he studies less than 121 minutes?

$X \sim N(\mu = 100,\ \sigma = 30)$

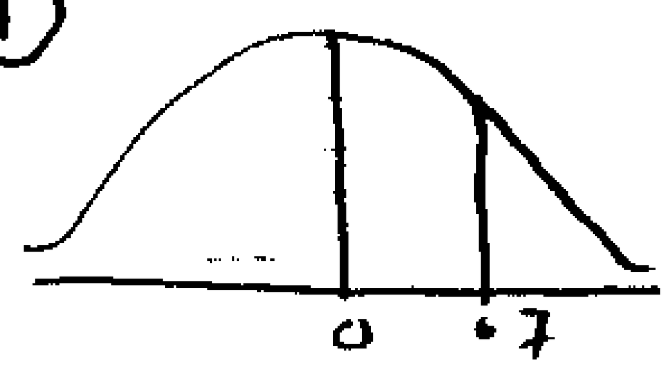
$$p(X < 121) = p\left(\frac{X-100}{30} < \frac{121-100}{30}\right) \quad 1$$

$$= p(Z < 0.7)$$

$$= 0.5000 + p(0 \le Z \le .7)$$

$$= 0.5000 + 0.2580 \rightarrow 1$$

$$= 0.7580 \quad 1$$



b. If students who typically obtain A+ grades in their courses study at least 180 minutes [redacted] daily, what is the percentage of these KFUPM students?

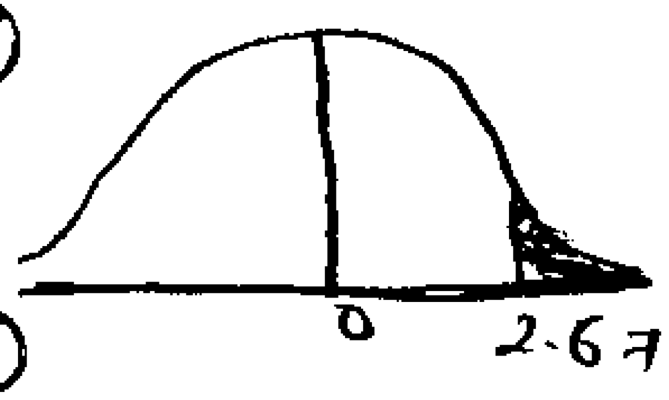
$$p(X \ge 180) = p\left(\frac{X-100}{30} > \frac{180-100}{30}\right) \quad 1$$

$$= p(Z > 2.67)$$

$$= 0.5 - p(0 \le Z \le 2.67) \quad 1$$

$$= 0.5 - 0.4962 = 0.0038 \quad \Big\} \ 2$$

or, Their percentage $= (.0038)(100\%) = 0.38\%$



c. Find x where 82% of the students study less than x minutes [redacted]

$$p(X \le x) = .8200 \quad \} \ 1$$

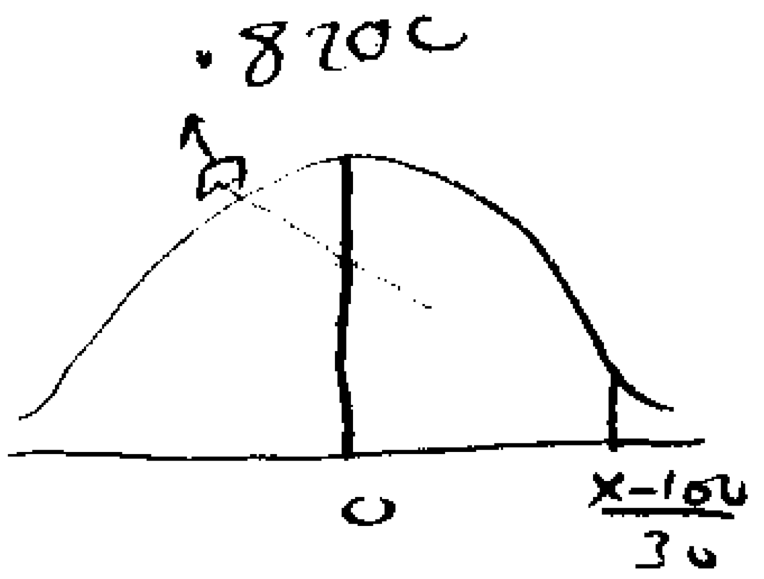
$$p\left(\frac{X-100}{30} \le \frac{x-100}{30}\right) = .8200 \quad \} \ 1$$

$$p\left(Z \le \frac{x-100}{30}\right) = .8200$$

$$\Rightarrow p\left(0 \le Z \le \frac{x-100}{30}\right) = .8200 - .5 = .3200$$

$$\Rightarrow \frac{x-100}{30} = .92 \ \Rightarrow\ x = (.92)(30) + 100 = 127.6 \quad \} \ 1$$

1



d. If 8 KFUPM students are selected at random, then find the probability that at most one of them will study less than 121 minutes.

In this case we have a binomial dist. with $n = 8$ 1

and p: prob. of success $= p(X < 121) = .758$ (from part (a))

1 $\Rightarrow q = 1 - .758 = .242$

$$p(\text{At most one}) = p(X \le 1)$$

$$= p(0) + p(1) \quad \} \ 1$$

$$= C_0^8 (.758)^0 (.242)^8 + C_1^8 \cdot (.758)^1 (.242)^7$$

$$= 0.000011763 + 0.000294758 \quad \Big\} \ 1$$

$$= 0.00031$$

Question .5 (3+2=5-Points) * Solutions - A-5

The percentage of students who will be admitted to the university after taking an entrance exam is 62%. A random sample of 9 students from those who took the entrance exam is selected. Then:

a. Find the probability that 3 from them will be admitted to the university.

$X \sim$ binomial with $n = 9$, $p = 0.62 \Rightarrow q = 1 - .62 = 0.38$ 1

$$p(X=3) = C_3^9 \cdot (.62)^3 (.38)^6 = 0.0603 \quad \Big\} \ 2$$

b. Find the expected number of students in the sample who will be admitted to the university.

$$E(X) = np = (9)(.62) = 5.58 \quad \Big\} \ 2$$

Question .6(4+4+4=12-Points)

Suppose that on the average there are 3 car accidents weekly at the 4th street. Then:

a. Find the probability that there will be at most 2 car accidents at the 4th street next week.

$\lambda = 3$, $t = 1$

$$p(\text{At most } 2) = p(X \le 2) = p(0) + p(1) + p(2) \quad \Big\} \ 1$$

$$= \frac{(3)^0 e^{-3}}{0!} + \frac{(3)^1 e^{-3}}{1!} + \frac{(3)^2 e^{-3}}{2!} = \quad \Big\} \ 2$$

$$= 0.0498 + 0.1494 + 0.2240 = 0.4232 \quad \Big\} \ 1$$

b. Find the probability that there will be at least 1 car accident at the 4th street in the coming 2 weeks. $\lambda = 3$, $t = 2 \Rightarrow \lambda t = (3)(2) = 6$ } 1

$$p(\text{At least } 1) = p(X \ge 1) = 1 - p(X < 1) \quad \Big\} \ 1$$

$$= 1 - p(X=0) = 1 - \frac{(6)^0 e^{-6}}{0!} \quad \Big\} \ 1$$

$$= 1 - 0.0025 = 0.9975 \quad \Big\} \ 1$$

c. Find the **mean** and **standard deviation** of the number of car accidents in one year. (Hint: Use one year =53 weeks)

$\lambda = 3$, $t = 53$

The mean $= E(X) = \lambda t = (3)(53) = 159$ } 2

The standard deviation $= \sigma = \sqrt{\lambda t} = \sqrt{(3)(53)} = 12.6095$ } 2