SOLUTIONS

King Fahd University of Petroleum & Minerals
Department of Mathematical Science
STAT-211-Term052-I
Quiz #5 Section:

Name: ID: Serial:

Question One (4+4 = 8)

If the ratio of defective items in a shipment is 20%, a sample of size five is taken randomly without replacement, From 100 items, (N = 100).

a. Find the probability of at least one defective item.

D : Defective
G: Good

D	G
20	80

N = 100 → Sample n = 5

$$p(x \geq 1) = 1 - p(x < 1)$$
$$= 1 - p(x=0) = 1 - \frac{C_0^{20} \cdot C_5^{80}}{C_5^{100}} = 1 - .3193 = 0.6807. \Big\} \ 4$$

b. Find the probability all the sample items are defective.

$$p(x=5) = \frac{C_5^{20} \cdot C_0^{80}}{C_5^{100}} = 0.0002 \Big\} \ 4$$

Question Two (3+3=6)

a. The number of a customers served by an employee in a certain bank follow a Poisson distribution with an average of 6 customers per 30 minutes, find probability of serving eight customers in one hour.

$\lambda = 6$ per .5 hr , $t = 1$ hr $\Rightarrow \lambda t = (6)(2) = 12$

$$p(x = 8) = \frac{(12)^8 e^{-12}}{8!} = 0.0655 \Big\} \ 3$$

b. If X is uniformly distributed over the interval $[-5, 1]$, find $P(-3 \leq x \leq 2)$

$$f(x) = \begin{cases} \frac{1}{6} & , -5 \leq x \leq 1 \\ 0 & , \text{o.w} \end{cases}$$

-5 -3 1 2

$$p(-3 \leq x \leq 2) = p(-3 \leq x \leq 1) = \frac{1}{6}(1-(-3)) = \frac{4}{6} = \frac{2}{3} \Big\} \ 3$$

Question Three (6)

The yearly incomes for a group of 10,000 professional people is normally distributed with mean $\mu = \$50,000$ and standard deviation $\sigma = \$4000$, find the number of people who have a yearly income below \$44,000

$$X \sim N(\mu = 50{,}000 , \sigma = 4000)$$

$$p(x \leq 44{,}000) = p\left(\frac{X - 50000}{4000} \leq \frac{44000 - 50000}{4000}\right) = p(Z \leq -1.5) \Big\} \ 2$$

$$= p(Z > 1.5) = 0.5000 - p(0 \leq Z \leq 1.5) = .5000 - 0.4332 \Big\} \ 2$$
$$= 0.0668$$

$\Rightarrow$ The number of people who have a yearly incom below 4400 = (10 000)(.0 668) = 668 } 2

You may use this table:

z_0	0.2	0.5	1.5	2.0	2.2	2.25
$P(0 < Z < z_0)$	0.0793	0.1915	0.4332	0.4772	0.4861	0.4878

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STAT-211-Term052-II

Quiz #5 Section:

Name: ID: Serial:

Question One (4+4=8)

If the ratio of defective items in a shipment is 10%, a sample of size five is taken randomly with replacement, then:

a. Find the probability of at least one defective item.

$X \sim$ binomial with $n=5$, $p=.10$, $q=.90$

$$\left.\begin{aligned} p(X\geq 1) &= 1 - p(X<1) \\ &= 1 - p(X=0) = 1 - C^5_0 (.10)^0 (.90)^5 \\ &= 1 - .5905 = 0.4095 \end{aligned}\right\} 4$$

b. Find the mean and the variance of the number of defectives.

$$\left.\begin{aligned} \mu &= np = (5)(.10) = .5 \\ \sigma^2 &= npq = (5)(.10)(.90) = 0.45 \end{aligned}\right\} 4$$

Question Two (3+3=6)

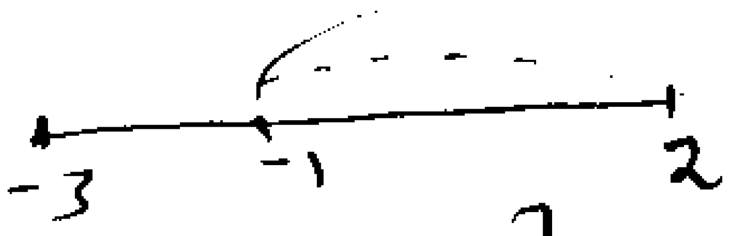
a. The number of a customers served by an employee in a certain bank follow a Poisson distribution with an average of five customers per hour, find probability of serving eight customers in 2 hours.

$\lambda = 5$ per 1 hr, $t=2 \Rightarrow \lambda t = (5)(2) = 10$

$$\left. p(X=8) = \frac{(10)^8 e^{-10}}{8!} = 0.1126 \right\} 3$$

b. If X is uniformly distributed over the interval $[-3, 2]$, find $P(X \geq -1)$

$$f(x) = \begin{cases} \frac{1}{5}, & -3 \leq x \leq 2 \\ 0, & \text{o.w} \end{cases}$$



$$\left. p(X\geq -1) = p(-1\leq X \leq 2) = \frac{1}{5}(2-(-1)) = \frac{3}{5} = 0.60 \right\} 3$$

Question Three (6)

The yearly incomes for a group of 20,000 professional people is normally distributed with mean $\mu = \$60,000$ and standard deviation $\sigma = \$5000$, find the number of people who have a yearly income over \$70,000

$X \sim N(\mu = 60000, \sigma = 5000)$

$$p(X > 7000) = p\left(\frac{X-60000}{5000} > \frac{70000-60000}{5000}\right) \quad 2$$

$$\left. = p(Z > 2) = 0.5 - p(0\leq Z \leq 2) \right\} 2$$

$$= 0.5000 - 0.4772 = 0.0228$$

$\therefore$ The number of people who have a yearly income over 70000 $= (20000)(.0228) = 456.$ $\}$ 2

You may use this table:

z_0	0.2	0.5	1.5	2.0	2.2	2.25
$P(0 < Z < z_0)$	0.0793	0.1915	0.4332	0.4772	0.4861	0.4878