Question.1. (6 Points)
Answer the following questions by indicating that each statement is either **True** or **False**.

1. If a company has the opportunity to bid on three contracts, A, B, and C, the sample space for the contracts that are awarded to the company is {A, B, C}:---True

2. One of the difficulties in using the relative frequency of occurrence method for assessing probabilities in business situations is getting a large enough set of examples that match the one in question. :---True

3. The random variable, number of customers entering a store between 9 AM and noon, is an example of a discrete random variable:---True

4. The distribution for the number of emergency calls to a city's 911 emergency number in a one-hour time period is likely to be described by a binomial distribution. :---False
   It is a Poisson

5. The primary application for the hypergeometric probability distribution is in situations where the sampling is done without replacement from a finite population. :---True

6. If the mean, median and mode are all equal for a continuous random variable, then the random variable is normally distributed. :--False

Question.2. (6 Points)
Answer the following questions by choosing the right answer.

1. Managers who are considering whether to order 1, 2, or 3 cases of a product will base their decision on the probability of selling a specified number of products. The probability would most likely be based on:

   a. Classical assessment.
   b. Subjective assessment.
   c. Relative frequency of occurrence.
   d. Both b and c could be used depending on the situation.

2. Which of the following statements is incorrect:

   a. The expected value of a discrete probability distribution is the long-run average value assuming the experiment will be repeated many times.
   b. The standard deviation of a discrete probability distribution measures the average variation of the random variable from the mean.
   c. The distribution is considered uniform if all the probabilities are equal.
   d. The mean of the probability distribution is equal to the square root of the expected value.

3. Which of the following is not a condition of the binomial distribution?

   a. Two possible outcomes for each trial
   b. The trials are independent
   c. The standard deviation is equal to the square root of the mean
   d. The probability of a success remains constant from trial to trial

4. If the number of defective items selected at random from a parts inventory is considered to follow a binomial distribution with n = 50 and p = 0.10, the expected number of defective parts is?

   a. 5.
   b. Approximately 2.24.
   c. More than 10.
   d. None of the above.

5. If a study is set up in such a way that a sample of people is surveyed to determine whether they have ever used a particular product, the likely probability distribution that would describe the random variable—the number who say yes—is a:

   a. Binomial distribution.
   b. Poisson distribution.
   c. Uniform distribution.
   d. Continuous distribution.

6. The hypergeometric probability distribution is used rather than the binomial or the Poisson when:
   a. The sampling is performed with replacement.
   b. The sampling is performed without replacement from an infinite population.
   c. The sampling is performed without replacement from a finite population.
   d. The sampling is performed with replacement from a finite population.

Question.3 (2+3+4 = 9 Points)
When a customer comes to a bank, there are three primary locations they may select to go to: teller, loan officer, or escrow department. Based on past experience, the following probability distribution applies:

Location	Probability
Teller	0 .60
Loan officer	0 .30
Escrow	0.10

Seventy percent of customers are **males**.
a. What is the probability that three consecutive customers all go to a teller?

p(The three go to a teller) = $(.6)(.6)(.6)$ by independence } ② points

$= .216$

b. Are the **two** events go to the Teller and go to Escrow mutually exclusive? Explain.

Let A : A customer go the Teller
B : " " " Escrow

A customer can not be on the teller and escrow on the same time, just one of them at a time.
$\therefore$ A and B are mutually exclusive.
and so $p(A \text{ and } B) = 0$.

③ points

c. What is the probability that the next customer will be male and will go to either the teller or the escrow department?

Let E_1 : The next customer will be male $\Rightarrow p(E_1) = .70$
E_2 : " " " " go to either teller or escrow

We want $p(E_1 \text{ and } E_2) = ?$

but $p(E_2) = p(A \text{ or } B)$ where A & B as in part (b) } ① point

$= p(A) + p(B) = .60 + .10 = .70$

$\therefore p(E_1 \text{ and } E_2) = p(E_1) \cdot p(E_2)$ because A male customer or Female customer may go to the teller or escrow, so E_1 and E_2 are indep.

① point for explaining

① point to formula

$= (.70)(.70)$

$= .49$ } ① point for answer

Question.4 (3 + 3+ 3 = 9 Points)
The number of no-shows for dinner reservations at the Al-Khobar Grille is a discrete random variable with the following probability distribution:

No-shows (x)	Probability
0	0.30
1	0.20
2	0.20
3	0.15
4	0.15

a. What is the expected number of no-shows?

$$E(X) = \sum_{\text{all } x} x\, P(x)$$
$$= (0)(.30) + (1)(.20) + (2)(.20) + (3)(.15) + (4)(.15)$$
$$= 0 + .20 + .40 + .45 + .60$$
② points for Calculations

$$= 1.65$$ ① point

b. Find the standard deviation of the no-show.

$$\sigma_x = \sqrt{\sum (x - E(x))^2 P(x)}$$
$$= \sqrt{(0-1.65)^2(.3) + (1-1.65)^2(.2) + (2-1.65)^2(.2) + (3-1.65)^2(.15) + (4-1.65)^2(.15)}$$
$$= \sqrt{0.81675 + .0845 + .0245 + 0.273375 + 0.828375}$$
$$= \sqrt{2.0275}$$
② points for Calculation

$$= 1.42390$$ ① point

c. The loss in the tip given to the Waters is a function of the no-shows given by 2X +1, what is the expected loss on the tip?

$$E(2X+1) = 2E(X) + 1$$
$$= 2(1.65) + 1$$
② points

$$= 4.3$$ ① point

Question.5 (3+3+3 = 9 Points)
A direct marketing company believes that the probability of making a sale when a call is made to an individual's home is 0.10. A sample **eight** houses is selected.

a. What is the probability that at most 2 sales will be made to theses eight houses?

Let X be the number of Sales made by a call, then

X has a binomial dist. with $n = 8$, $p = 0.10$, $q = 1 - p = 1 - .10 = .90$

$$\left.\begin{aligned} p(\text{At most } 2) &= p(X \le 2) \\ &= p(0) + p(1) + p(2) \\ &= C_0^8 (.1)^0 (.9)^8 + C_1^8 (.1)^1 (.9)^7 + C_2^8 (.1)^2 (.9)^6 \end{aligned}\right\} \text{② points for Calculations}$$

$$= .4305 + 0.3826 + 0.1488 = 0.9619 \quad \} \text{ ① point}$$

b. What is the probability that will be no sale made to theses eight houses?

$$\text{② points} \left\{\begin{aligned} &X \text{ is a binomial with } n = 8,\ p = \text{Prob. of no Sale} = .90,\ q = .1 \\ &p(X = 8) = C_8^8 (.9)^8 (.1)^0 = (1)(.9)^8 (1) \end{aligned}\right.$$

$$= (.9)^8 = 0.4305 \quad \} \text{ ① point.}$$

OR: By Considering X as the number of Sales $\Rightarrow$ $p = .1$, $q = .9$

$$\Rightarrow p(X = 0) = C_0^8 (.1)^0 (.9)^8 = (.9)^8 = .4305.$$

c. Find the average and the standard deviation of the sale.

$$\left.\begin{aligned} \text{I. The average} &= \mu_X = E(X) = np \\ &= (8)(.10) = 0.80 \end{aligned}\right\} \text{ ①.5 points}$$

$$\left.\begin{aligned} \text{II. The standard deviation} &= \sigma_X = \sqrt{npq} \\ &= \sqrt{(8)(.1)(.9)} \\ &= 0.8485 \end{aligned}\right\} \text{ ①.5 points.}$$

Question.6 (3+3= 6 Points)
A Carpet Company prides itself on high quality carpets. At the end of each day, the company quality managers select one square yards for inspection. The quality standard calls for an average of 0.5 defects per square yard.

a. What is the probability that at least 2 defectives will be found in the next 3 square yards selected?

a. $\lambda = .5$, $t = 3 \Rightarrow \overbrace{\lambda t = (.5)(3) = 1.5}^{\text{1 point}}$ (Poisson with $\lambda t = 1.5$)

$$\left.\begin{aligned} p(\text{At least } 2) &= p(X \geq 2) = 1 - p(X < 2) = 1 - p(X \leq 1) \\ &= 1 - (p(0) + p(1)) \\ &= 1 - \left(\frac{(1.5)^0 e^{-1.5}}{0!} + \frac{(1.5)^1 e^{-1.5}}{1!} \right) \end{aligned}\right\} \text{1 point}$$

$$= 1 - (.2231 + .3347) = 1 - .5578 = 0.4422 \Big\} \text{1 point.}$$

b. What is the probability that no defective will be found in a piece of carpet of size 10 square yards that you just bought from the product of this company.

Poisson: $\lambda = .5$, $t = 10 \Rightarrow \lambda t = (.5)(10) = 5 \Big\}$ 1 point

$p(\text{No defective}) = p(X=0) = \dfrac{(5)^0 e^{-5}}{0!} \Big\}$ 1 point

$= e^{-5} = 0.0067 \Big\}$ 1 point.

Question.7 (3+3 = 6 Points)
A company has **twenty** cars that are available for use by company executives for official business purposes. **Eight** of these cars are luxury type cars. If **five** cars are randomly selected:

a. What is the probability that exactly **one** care is luxury type?

Hypergeometric: $N = 20$, $n = 5$, $X = 8$, x: number of luxuray cars on the sample.

2 points $\Big\{ p(X=1) = \dfrac{C_1^8 \cdot C_4^{12}}{C_5^{20}} = \dfrac{(8)(495)}{15504}$

$= 0.2554 \Big\}$ 1 point.

b. What is the expected number of luxury type car?

$E(\text{luxuray type}) = n \cdot p$, $p = \text{Prob. of luxury Car} = \dfrac{8}{20} = \dfrac{2}{5} \Big\}$ 1 point

$= (5)\left(\frac{2}{5}\right) \Big\}$ 1 point

$= 2 \Big\}$ 1 point.

Note: The Student can find the prob. dist. of X, and then $E(X) = \sum x\, p(x)$ as follows:

x	0	1	2	3	4	5
$P(x)$	0.0511	0.2554	0.3973	0.2384	0.0542	0.0036

$E(X) = \sum x\, p(x) = (0)(.0511) + (1)(.2554) + \cdots + (5)(.0036) = 2.0$

Question.8 (3+3+3 = 9 Points)
The vehicle speeds on a city street have been determined to be normally distributed with a mean of 33.2 mph and a variance of 16.

a. What is the probability that a randomly selected vehicle its speed will be between 30 and 35 mph?

Vechile speeds $\Rightarrow X \sim N(\mu = 33.2, \sigma^2 = 16)$

$p(30 < X < 35) = p\left(\frac{30-33.2}{4} < \frac{X-33.2}{4} < \frac{35-33.2}{4}\right)$ } ① point

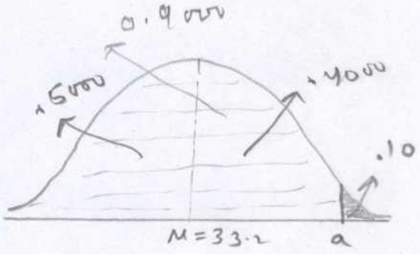
$= p(-0.8 < Z < 0.45)$

$= p(0 \le Z < .8) + p(0 < Z \le .45)$

$= 0.2881 + 0.1736$ } ① point for areas

$= 0.4617.$ } ① point

b. What is cutoff point of the upper 10% of the speed?



Let a be the Cutoff point of the upper 10%

$p(X > a) = .10 \Rightarrow p(X \le a) = 1 - .1 = .90$

$p\left(\frac{X-33.2}{4} \le \frac{a-33.2}{4}\right) = .90$ } ① point

$p\left(0 \le Z \le \frac{a-33.2}{4}\right) = .4000$

From the standard normal table $\Rightarrow \frac{a-33.2}{4} = \frac{1.28}{1}$ } ① point

$\Rightarrow a - 33.2 = (4)(1.28) = 5.12$

$\therefore a = 5.12 + 33.2 = 38.32$ } ① point.

c. What is the probability that if **three** randomly selected vehicles, the speed of **two** of them will exceed the 35 mph?

Let Y be the number of Cars whose speed exceed 35 mph.

then $Y \sim$ binomial with $n = 3$, $p = p(X > 35)$

$p(X > 35) = p\left(\frac{X-33.2}{4} > \frac{35-33.2}{4}\right)$

$= p(Z > .45) = 0.5 - .1736 = 0.3264$ } ① point

$\therefore p(Y=2) = C_2^3 (.3264)^2 (1 - .3264)^1$ } ① point

$= (3)(.3264)^2(.6736)$

$= 0.2153.$ } ① point