

Q3. (8 marks)

The following table is a partial probability distribution for some company's projected profits (X = profit in SR 1000s) for the first year of operation.

(The negative value denotes a loss).

x	-100	0	50	100	150	200
P(x)	0.1	0.2	0.3	0.25	0.1	

- What is the proper value for $P(X = 200)$?
- What is the probability that the company will be profitable?
- What is the probability that the company will make at least SR 100,000?
- What is the expected profit for the company?

$$\textcircled{2} \text{ a. } P(X=200) = 1 - [P(X=-100) + P(X=0) + P(X=50) + P(X=100) + P(X=150)]$$

$$= 0.05$$

$$\textcircled{2} \text{ b. } P(\text{Company is profitable}) = P(X \geq 50)$$

$$= 0.70$$

$$\textcircled{2} \text{ c. } P(X \geq 100,000) = 0.25 + 0.1 + 0.05 = 0.40$$

$$\textcircled{2} \text{ d. } \begin{aligned} \text{Expected Profit} &= (-100)(0.1) + (0)(0.2) + 50(0.3) + \\ &\quad (100)(0.25) + 150(0.1) + 200(0.05) \end{aligned}$$

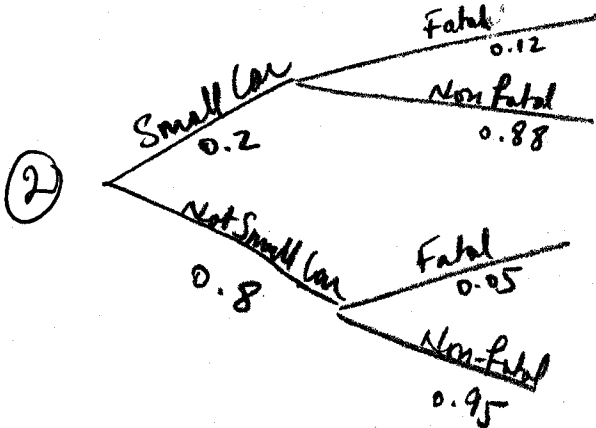
$$= -10 + 15 + 25 + 15 + 10$$

$$\textcircled{1} \text{ Expected Profit} = \text{SR } 55,000$$

Q4. (6 Marks)

Small cars constitute 20% of vehicles on the road. The percentage of accidents involving small cars leading to a fatality (death) is 12%, and the percentage of accidents NOT involving small cars leading to a fatality is 5%.

- What is the probability that an accident will not lead to a fatality?
- Suppose that a reported accident have led to a fatality, what is the probability that a small car was involved?



$$a. P(\text{Non-fatal accident})$$

$$= (0.2)(0.88) + (0.8)(0.95)$$

$$= \frac{0.176}{\quad} + \frac{0.76}{\quad}$$

$$\textcircled{2} = \textcircled{0.936}$$

$$\textcircled{2} \text{ b. } P(\text{Small Car} | \text{Fatal Accident})$$

$$= \frac{P(\text{Small Car} \& \text{Fatal Accident})}{P(\text{Fatal Accident})} = \frac{(0.2)(0.12)}{0.064}$$

$$= \frac{0.024}{0.064} = \textcircled{0.375}$$

Q5. (6 Marks)

Historical data show that the average number of patient arrivals at the intensive care unit of General Hospital is 3 patients every 2 hours. Assume that the patient arrivals are distributed according to Poisson distribution. Determine the probability of 6 patients arriving in a five-hour period.

$$\textcircled{2} \left\{ \begin{array}{l} \lambda = 3 \text{ patients} / 2 \text{ hrs} \\ = 1.5 \text{ patients} / \text{hr.} \end{array} \right.$$

$$\textcircled{3} P(\# \text{ of patients is } 6 \text{ in } 5 \text{ hours}) = \frac{e^{-(1.5)5} (1.5)^6}{6!}$$

$$\textcircled{1} \longrightarrow = 0.1367$$

Q6. (9 Marks)

Suppose that we have a sample space $S = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}$ with the following probabilities:

$$P(E_1) = 0.05,$$

$$P(E_2) = P(E_3) = 0.20,$$

$$P(E_4) = 0.25$$

$$P(E_5) = 0.15,$$

$$P(E_6) = 0.10,$$

$$P(E_7) = 0.05$$

and let $A = \{E_1, E_4, E_6\}$ and $B = \{E_2, E_4, E_7\}$.

- Find $P(A)$ and $P(A \cap B)$.
- Find $P(\bar{B})$ and $P(A \cup B)$.
- Are the events A and B mutually exclusive? Explain.

$$a. P(A) = P(E_1) + P(E_4) + P(E_6)$$

$$\textcircled{2} = 0.05 + 0.25 + 0.1$$

$$= \textcircled{0.40}$$

$$\textcircled{1} P(A \cap B) = P(E_4) = \textcircled{0.25}$$

$$b. P(\bar{B}) = P(\{E_1, E_3, E_5, E_6\})$$

$$\textcircled{2} = 0.05 + 0.20 + 0.15 + 0.10$$

$$= \textcircled{0.50}$$

$$\text{OR } P(\bar{B}) = 1 - P(B)$$

$$= 1 - [0.2 + 0.25 + 0.05]$$

$$= 1 - 0.50$$

$$= \textcircled{0.50}$$

$$\textcircled{2} P(A \cup B) = P(\{E_1, E_2, E_4, E_6, E_7\})$$

$$= 1 - [P(E_3) + P(E_5)]$$

$$= 1 - (0.2 + 0.15) = \textcircled{0.65}$$

c. A and B are not mutually exclusive, since

$$\textcircled{2} A \cap B = E_4 \neq \emptyset.$$

Q7. (6 Marks)

Weekly demand at a grocery store for a brand of breakfast cereal is normally distributed with a mean of 800 boxes and a standard deviation of 75 boxes.

- What is the probability that weekly demand is between 725 and 950 boxes?
- The store orders cereal from a distributor weekly. How many boxes should the store order for a week to have only a 5% chance of running short of this brand of cereal during the week?

$X = \text{weekly demand} ; X \sim N(800, (75)^2)$

$$P(725 \leq X \leq 950) = P\left(\frac{725-800}{75} \leq \frac{X-800}{75} \leq \frac{950-800}{75}\right) \quad \text{where } Z \sim N(0,1)$$

$$= P(-1 \leq Z \leq 2)$$

$$= 0.4772 + 0.3413$$

$$= 0.8185$$

Find x_0 such that $P(X \geq x_0) = 0.05$

$$P\left(\frac{X-800}{75} \geq \frac{x_0-800}{75}\right) = 0.05$$

$$\text{Since } P(Z \geq 1.645) = 0.05$$

$$\Rightarrow \frac{x_0-800}{75} = 1.645 \Rightarrow x_0 = (1.645)(75) + 800 = 923.375$$

Q8. (5 Marks)

There are 8 flights daily from Riyadh to Dammam. The probability that any flight arrives late is 0.20, and that flight arrivals are independent.

- What is the probability that at least 2 flights arrive late?
- How many flights do we expect to arrive late daily?

$X = \# \text{ of flights arriving late} ; X \sim B(8, 0.20)$

$$a. P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - [P(X=0) + P(X=1)]$$

$$= 1 - \left[\binom{8}{0} (0.2)^0 (0.8)^8 + \binom{8}{1} (0.2)^1 (0.8)^7 \right]$$

$$= 1 - (0.167 + 0.335) = 0.498$$

$$b. \text{Expected Number of late flights} = (0.2)(8) = 1.6 \text{ fly}$$