

**4.2.**

- a.  $P(\text{Brown}) = \# \text{ Brown} / \text{Total} = 310 / 982 = 0.3157$
- b.  $P(\text{YZ-99}) = \# \text{ YZ-99} / \text{Total} = 375 / 982 = 0.3819$
- c.  $P(\text{YZ-99 and Brown}) = 205 / 982 = 0.2088$
- d. The two events are not mutually exclusive since their joint probability is 0.1324. It is possible to choose a White YZ-99 product. To be mutually exclusive the outcome for one event (e.g., White) means the other outcome (YZ-99) cannot occur.

**4.9.**

<b>Origination</b>	<b>Early</b>	<b>On-Time</b>	<b>Late</b>	<b>Total</b>
San Francisco	25	50	100	<b>175</b>
Los Angeles	50	100	75	<b>225</b>
<b>Total</b>	<b>75</b>	<b>150</b>	<b>175</b>	<b>400</b>

- a.  $P(\text{Early}) = 75 / 400 = 0.1875$ .
- b.  $P(\text{Los Angeles}) = 225 / 400 = 0.5625$ .
- c.  $P(\text{Early Given Los Angeles}) = 50 / 225 = 0.2222$ .  $P(\text{Early}) = 75 / 400 = 0.1875$
- d. Let E = Early, O= On-Time, and L = Late. The events are as follows:

<b>Elementary Event</b>	<b>Flight 1</b>	<b>Flight 2</b>	<b>Flight 3</b>
Event 1	E	E	E
Event 2	E	E	O
Event 3	E	E	L
Event 4	E	O	E
Event 5	E	O	O
Event 6	E	O	L
Event 7	E	L	E
Event 8	E	L	O
Event 9	E	L	L
Event 10	O	E	E
Event 11	O	E	O
Event 12	O	E	L
Event 13	O	O	E
Event 14	O	O	O
Event 15	O	O	L
Event 16	O	L	E
Event 17	O	L	O
Event 18	O	L	L
Event 19	L	E	E
Event 20	L	E	O
Event 21	L	E	L
Event 22	L	O	E
Event 23	L	O	O
Event 24	L	O	L
Event 25	L	L	E
Event 26	L	L	O
Event 27	L	L	L

The sample space consists of the 27 elementary events listed in the table. Sample Space = {Event 1, Event 2, Event 3, ..., Event 27}

#### 4.15.

Students can use Excel's pivot table feature to answer these questions.

a.  $P(\text{Atlanta}) = 24/110 = 0.2182$

Manufacturing Plant	Total
Boise	78
Atlanta	24
Reno	8
Grand Total	110

b.  $P(\text{Wiring}) = 23/110 = 0.2091$

Complaint Code	Total
Corrosion	35
Cracked Lens	45
Wiring	23
Sound	7
Grand Total	110

c.  $P(\text{Atlanta and Wiring}) = 8/110 = 0.0727$

Complaint Code	Manufacturing Plant			Grand Total
	Boise	Atlanta	Reno	
Corrosion	30	3	2	35
Cracked Lens	31	11	3	45
Wiring	13	8	2	23
Sound	4	2	1	7
Grand Total	78	24	8	110

d.  $P(\text{Day Shift and Atlanta and Cracked Lens}) = 8/110 = 0.0727$

Complaint Code	Manufacturing Plant		
	Atlanta		
	Day	Swing	Graveyard
Corrosion	1	1	1
Cracked Lens	8	2	1
Wiring	5	2	1
Sound	2		
Grand Total	16	5	3

- e. The most likely profile would be the largest number which would be the Boise day shift for cracked lens

	Manufacturing Plant							
	Boise			Atlanta			Reno	
Complaint Code	Day	Swing	Grave-yard	Day	Swing	Grave-yard	Day	Swing
Corrosion	20	10		1	1	1	2	
Cracked Lens	21	10		8	2	1	3	
Wiring	10	1	2	5	2	1	1	1
Sound	2	2		2				1
Grand Total	53	23	2	16	5	3	6	2

#### 4.16.

- a.  $P(A) = 1,000/2,100 = 0.4762$
- b.  $P(A \text{ and } B) = 0$  since A and B are mutually exclusive
- c.  $P(B \text{ and } F) = 300/2100 = 0.1429$
- d.  $P(E|A) = 600/1000 = 0.60$
- e.  $P(A \text{ or } F) = P(A) + P(F) - P(A \text{ and } F)$   
 $= 1000/2100 + 900/2100 - 300/2100$   
 $= 0.4762 + 0.4286 - 0.1429$   
 $= 0.7619$

#### 4.35.

There are a total of 6 companies.

- a.  $P(\text{Ace}) = 1/6 = 0.1667$
- b.  $P(\text{Win1 and Win2}) = (1/6)(1/6) = 0.0278$
- c.  $P(\text{Lose1 and Lose2}) = (5/6)(5/6) = 0.6944$
- d.  $P(\text{Win1 and Lose2}) + P(\text{Lose1 and Win2}) = (1/6)(5/6) + (5/6)(1/6) = 0.2778$
- e.  $P(\text{Win1 and Lose2}) + P(\text{Lose1 and Win2}) + P(\text{Win1 and Win2}) = 0.1389 + 0.1389 + 0.0278 = 0.3056$  or  $1 - P(\text{Lose 1 and Lose 2}) = 1 - 0.6944 = 0.3056$

**4.39.**

Students can use Excel's pivot table feature to answer this question.

a.  $P(\text{Wiring/Atlanta}) = P(\text{Wiring and Atlanta})/P(\text{Atlanta}) = (8/110)/(24/110) = 0.3333$

Complaint Code	Manufacturing Plant			Grand Total
	Boise	Atlanta	Reno	
Corrosion	30	3	2	35
Cracked Lens	31	11	3	45
Wiring	13	8	2	23
Sound	4	2	1	7
Grand Total	78	24	8	110

b. Using the table from above and the information in the problem you know:

$P(\text{Boise/return}) = 78/110 = 0.7091$

$P(\text{Atlanta/return}) = 24/110 = 0.2182$

$P(\text{Reno/return}) = 8/110 = 0.0727$

Assuming the sample represents the population of returned products, the cost assignment should be based on the conditional probability of a city given that the product was returned. Thus, Boise will get 70.91% of the cost, Atlanta will get 21.82% and Reno will get 7.27% regardless of production volume

**4.47.**

To find the correlation, use equation 4-18:

$$\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

The covariance is found by equation 4-17:

x	P(x)	xP(x)	x - E(x)	y	P(y)	yP(y)	y - E(y)	[x - E(x)][y - E(y)]	P(xy)	[x - E(x)][y - E(y)]P(xy)
100	0.25	25	-125	500	0.25	125	85	(10,625.00)	0.10	(1,062.50)
200	0.40	80	-25	300	0.40	120	-115	2,875.00	0.50	1,437.50
300	0.20	60	75	400	0.20	80	-15	(1,125.00)	0.30	(337.50)
400	0.15	60	175	600	0.15	90	185	32,375.00	0.10	3,237.50
		225				415				3,275.00

The covariance is 3,275      The relationship between the two variables is positive

The standard deviation for variable x is found as follows:

x	P(x)	xP(x)	x-E(x)	[x-E(x)] <sup>2</sup>	[x-E(x)] <sup>2</sup> P(x)
100	0.25	25	-125	15625	3906.25
200	0.4	80	-25	625	250
300	0.2	60	75	5625	1125
400	0.15	60	175	30625	4593.75
		225			9875

$\sigma_x = \sqrt{9,875} = 99.37$

The standard deviation for y is found using:

y	P(y)	yP(y)	y - E(y)	[y-E(y)] <sup>2</sup>	[y-E(y)] <sup>2</sup> P(y)
500	0.25	125	85	7,225	1,806.25
300	0.40	120	-115	13,225	5,290.00
400	0.20	80	-15	225	45.00
600	0.15	90	185	34,225	5,133.75
		415			12,275.00

$$\sigma_y = \sqrt{12,275} = 110.79$$

The correlation is:  $\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = (3,275) / [(99.37)(110.79)] = 0.2975$

The correlation measures the strength of the linear relationship between the two variables. In this case, there is a weak positive linear relationship.

#### 4.50.

In order to calculate the mean, variance, and standard deviation, students will need to assume that all values occur at the midpoint. This is calculated by adding the upper and lower limit of each class and dividing by 2. The students will then determine the probabilities by dividing each frequency by the total of the frequencies.

a.

X	P(x)	xP(x)	x-E(x)	[x-E(x)] <sup>2</sup>	[x-E(x)] <sup>2</sup> P(x)
5	0.10	0.50	-21.70	470.890	47.0890
15	0.17	2.55	-11.70	136.890	23.2713
25	0.35	8.75	-1.70	2.890	1.0115
35	0.22	7.70	8.30	68.890	15.1558
45	0.16	7.20	18.30	334.890	53.5824
		26.70			140.1100

b.  $E(\text{cars}) = 26.70$

c.  $\text{Var}(\text{cars}) = 140.1100$ ;  $\text{St.Dev.}(\text{Cars}) = 11.8368$

d. To calculate this question the students need to determine the cumulative probability.

x	P(x)	Cum.P(x)
5	0.10	0.10
15	0.17	0.27
25	0.35	0.62
35	0.22	0.84
45	0.16	1.00

Student answers may vary but students should use x=45 since 35 is less than 85% and the question wants at least 85%. Assuming students use x=45 the following calculations will be made. If it takes 20 minutes per car a set of 2 employees can wash 3 cars per hour. Assuming an

eight hours day you would average  $45/8 = 5.625$  cars per hour. This means you would need two sets of employees to do 6 cars per hour. This means you would need 4 employees.

**4.63.**

- a.  $P(\text{win}) = 1/500 = 0.002$
- b.  $P(\text{win}) = 3/500 = 0.006$
- c. The probability assessment method used in a and b is the classical probability approach.

**4.74.**

- a. Student can initially fill out this part of the joint probability distribution table.

	Ski	Not Ski	
<b>Children not 8-16</b>			0.25
<b>Children 8-16</b>			0.35
<b>No Children</b>			
	0.4		

$$P(A1 \text{ and } B2) = P(A1|B2)P(B2) = 0.70(0.35) = 0.245$$

$$P(A1 \text{ and } B1) = P(A1|B1)P(B1) = 0.30(0.25) = 0.075$$

	Ski	Not Ski	
<b>Children not 8-16</b>	0.075		0.25
<b>Children 8-16</b>	0.245		0.35
<b>No Children</b>			
	0.4		

Students can now fill in the rest of the table by using the knowledge that to ski or not ski must sum to 1 and that the children categories must sum to 1. The remaining probabilities inside the table must add to the outside probabilities.

	Ski	Not Ski	
<b>Children not 8-16</b>	0.075	0.175	0.25
<b>Children 8-16</b>	0.245	0.105	0.35
<b>No Children</b>	0.080	0.320	0.40
	0.400	0.600	1.00

- b.  $P(\text{Ski and Children not 8-16}) = 0.075$
- c.  $P(\text{not ski} | \text{Children 8-16}) = 0.105/0.35 = 0.3$
- d. If the categories “skiing” and “family composition” are independent the product of the marginal probabilities should equal the joint probability. For example the probability of skiing times the probabilities of children not 8-16 should equal the joint probability of skiing and children not 8-16

Since  $.4(25) \neq .075$ ; the events are not independent