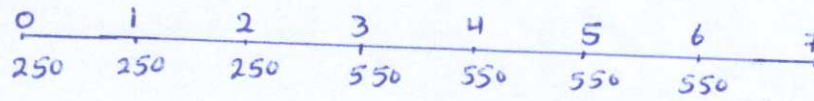


Question 3 : (7 Points)

- a. Find the present value of an annuity of \$250 due at the beginning of each year for three years, and \$550 due thereafter at the beginning of each year for four years. If the interest rate is 5% compounded annually? (3 points)

Solution: Method 1, $r = .05$



$$\begin{aligned} \text{The present values of all payments} &= 250 + 238.095 + 226.757 \\ &= 250(1.05)^0 + 250(1.05)^{-1} + 250(1.05)^{-2} \\ &\quad + 550(1.05)^{-3} + 550(1.05)^{-4} + 550(1.05)^{-5} + 550(1.05)^{-6} \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \textcircled{2}$$

$$= \$2483.81 \quad \left. \right\} \textcircled{1}$$

Method 2:

OR

$$\begin{aligned} \text{The present values of all payments} &= 550(1 + a_{\overline{7}|.05}) - 300(1 + a_{\overline{3}|.05}) \\ &= 550(1 + a_{\overline{7}|.05}) - 300(1 + a_{\overline{3}|.05}) \\ &\approx 550(1 + 5.075692) - 300(1 + 1.859414) \\ &\approx \$2483.81 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \textcircled{1}$$

$$\approx \$2483.81 \quad \left. \right\} \textcircled{1}$$

- b. In 12 years a \$42,000 bus will have a salvage value 20% of its cost. A new bus at that time is expected to sell for \$48,000. In order to provide funds for the difference between the replacement cost and the salvage value, a sinking fund is setup into which equal payments are placed at the end of each year. If the fund earns 6% compounded annually, how much should each payment be? (4 points)

Solution: Time = $n = 12$, $r = .06$, $R = ?$

$$\text{Salvage Value} = (0.20)(42000) = \$8,400 \quad \left. \right\} \textcircled{1}$$

$$\begin{aligned} \text{The amount needed after 12 years} &= 48000 - 8400 \\ &= 39,600 = S \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \textcircled{1}$$

$$S = R \cdot s_{\overline{n}|r} \Rightarrow R = \frac{S}{s_{\overline{n}|r}} = \frac{39600}{s_{\overline{12}|.06}} \quad \left. \right\} \textcircled{1}$$

$$\approx \frac{39600}{16.869941} = \$2,347.37 \quad \left. \right\} \textcircled{1}$$

OR

$$R = \frac{S}{s_{\overline{n}|r}} \quad \text{where} \quad s_{\overline{n}|r} = \frac{(1+r)^n - 1}{r} = \frac{(1.06)^{12} - 1}{.06} = 16.869942$$

$$\therefore R = \frac{39,600}{16.869942} = \$2,347.37$$