

Question 6: (10 Points)

Use **simplex method** to solve the following linear programming problem:

Maximize:

$$Z = 10x_1 + 15x_2 + 22x_3$$

Subject to :

$$x_1 + 2x_2 + 2x_3 \leq 40$$

$$-x_1 - x_2 - 2x_3 \geq -34$$

$$x_1, x_2, x_3 \geq 0$$

Change to :
 $\Rightarrow x_1 + x_2 + 2x_3 \leq 34$ } ①

The Standard Form:

$$x_1 + 2x_2 + 2x_3 + s_1 = 40$$

$$x_1 + x_2 + 2x_3 + s_2 = 34$$

$$-10x_1 - 15x_2 - 22x_3 + Z = 0$$

Table (1)

	x_1	x_2	x_3 (entering variable)	s_1	s_2	Z	b
s_1	1	2	2	1	0	0	40
s_2 (Departing Variable)	1	1	2	0	1	0	34
	-10	-15	-22	0	0	1	0

Quotients

$$40 \div 2 = 20$$

$$34 \div 2 = 17$$

Table (2)

$\frac{1}{2}R_2$
 $-R_2 + R_1, 11R_2 + R_3$

	x_1	x_2 (entering variable)	x_3	s_1	s_2	Z	b
s_1 (Departing Variable)	0	1	0	1	-1	0	6
x_3	$\frac{1}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	17
Z	1	-4	0	0	11	1	374

Quotients

$$6 \div 1 = 6$$

$$17 \div \frac{1}{2} = 34$$

Table (3)

$-\frac{1}{2}R_1 + R_2$
 $4R_1 + R_3$

	x_1	x_2	x_3	s_1	s_2	Z	b
x_2	0	1	0	1	-1	0	6
x_3	$\frac{1}{2}$	0	1	$-\frac{1}{2}$	1	0	14
Z	1	0	0	4	7	1	398

All indicators are non negative.

\therefore The maximum value of Z is 398 when

$$x_1 = 0, x_2 = 6, x_3 = 14.$$