

Question 4 : (10 Points)

A manufacturer of a certain product sells all that is produced. If the product is sold \$16 per unit, fixed cost is \$10,000 and variable cost \$8 per unit. Then

- a) Determine the break-even point (6 Points)

$$\text{The price} = \$16/\text{unit}, \text{F.C.} = \$10,000, \text{V.C.} = \$8/\text{unit}.$$

Let q_f be the number of units

At the break-even point \Rightarrow Total revenue = Total cost. ①

$$\Rightarrow ① 16q_f = 10,000 + 8q_f \Rightarrow 8q_f = 10,000 \Rightarrow q_f = 1250. ②$$

T.R. = $16(1250) = \$20,000 \Rightarrow$ The break-even point $(1250, \$20,000)$ ②

- b) Find the level of production at the break-even point if the total cost increases by 5%.

$$\text{The new T.C.} = (10,000 + 8q_f) + .05(10,000 + 8q_f) = 10,500 + 8.4q_f. ④ \text{ Points}$$

At the break-even point $T.R = T.C$

$$\Rightarrow ① 16q_f = 10,500 + 8.4q_f \Rightarrow 7.6q_f = 10,500 \Rightarrow q_f = \frac{10,500}{7.6} = 1381.58 \approx 1382. ①$$

The level of production ≈ 1382 units.

Question 5 : (10 Points)

Solve the following linear programming problem geometrically:

Minimize:

$$Z = 2x + 1.25y$$

Subject to :

$$\begin{aligned} 3x + y &\geq 9 \Rightarrow y_1 = -3x + 9 & \begin{array}{c|c} x & 0 \\ \hline y & 9 \\ \hline \end{array} & \begin{array}{c|c} x & 0 \\ \hline y & 0 \\ \hline \end{array} \\ 4x + 3y &\geq 22 \Rightarrow y_2 = -\frac{4}{3}x + \frac{22}{3} & \begin{array}{c|c} x & 0 \\ \hline y & \frac{22}{3} \\ \hline \end{array} & \begin{array}{c|c} x & 0 \\ \hline y & 0 \\ \hline \end{array} \\ x + 2y &\geq 8 \Rightarrow y_3 = -\frac{1}{2}x + 4 & \begin{array}{c|c} x & 0 \\ \hline y & 4 \\ \hline \end{array} & \begin{array}{c|c} x & 0 \\ \hline y & 0 \\ \hline \end{array} \\ x, y &\geq 0 \end{aligned}$$

$$① A = (8, 0)$$

$$② D = (0, 9)$$

B : The intersection point

between y_2 & y_3

$$-\frac{4}{3}x + \frac{22}{3} = -\frac{1}{2}x + 4$$

$$-\frac{4}{3}x + \frac{1}{2}x = 4 - \frac{22}{3}$$

$$-\frac{5}{6}x = -\frac{10}{3}$$

$$\Rightarrow -15x = -60 \Rightarrow x = 4$$

$$y = -\frac{1}{2}(4) + 4 = -2 + 4 = 2$$

$$① \therefore B = (4, 2)$$

C : The intersection between y_1 & y_2

$$-3x + 9 = -\frac{4}{3}x + \frac{22}{3}$$

$$-\frac{5}{3}x = -\frac{5}{3} \Rightarrow x = 1, y = 6$$

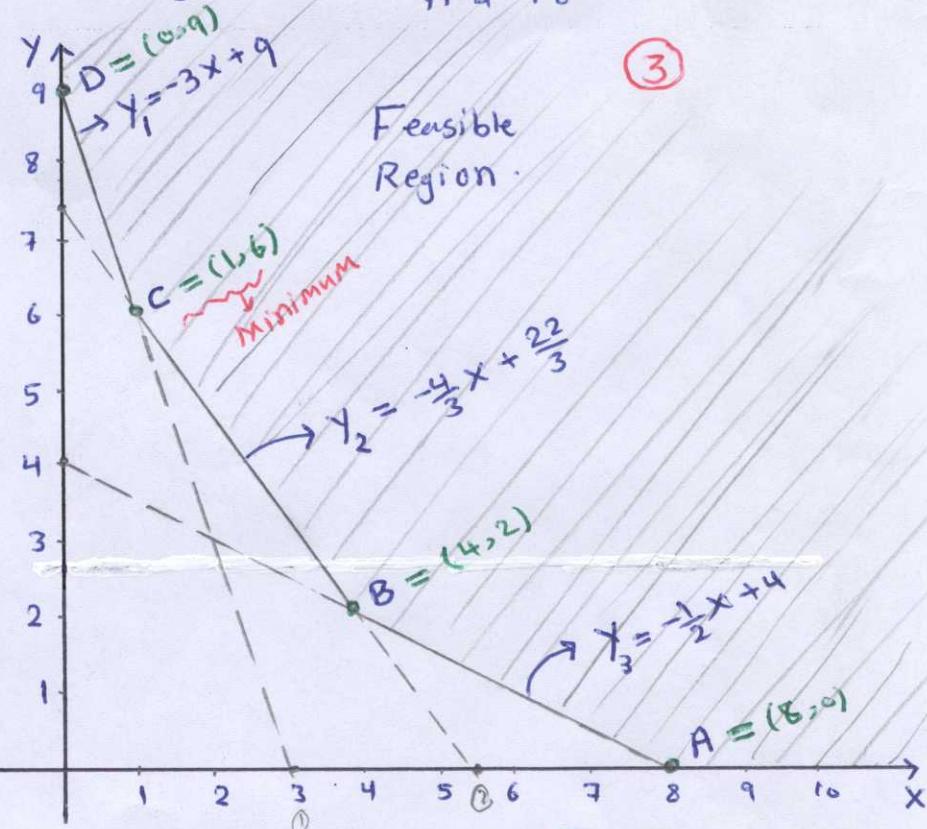
$$① \therefore C = (1, 6) \Rightarrow Z(A) = 2(8) + 1.25(0) = 16 + 0 = 16$$

$$Z(B) = 2(4) + 1.25(2) = 8 + 2.5 = 10.5$$

$$Z(C) = 2(1) + 1.25(6) = 2 + 7.5 = 9.5 \rightarrow \text{minimum}$$

$$Z(D) = 2(0) + 1.25(9) = 0 + 11.25 = 11.25$$

\therefore The minimum value of Z is 9.5 at $C = (1, 6)$ ①



$$\left. \begin{array}{l} ② \\ ③ \end{array} \right\}$$

minimum