# King Fahd University of Petroleum and Minerals
## College of Sciences
## Prep-Year Math Program

**Key**

**Math 001 Exam II**
**Term 022 (2002-2003)**
**Saturday, April 26, 2003**
**Time Allowed: 90 Minutes**

**Key**

**Student's Name:** ..........................................................................
**ID #:** .................................. **Section #:** ..........................

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**This exam consists of two parts:**

**Part I : Multiple Choice** **Bubble the correct answer on the OMR sheet**
**Part II : Written** **Provide neat and complete solutions.**
**Show all necessary steps for full credit.**

**Calculators, pagers, or mobiles are NOT allowed during this examination.**

| Question | Points | Student's Score | Grader |
|---|---|---|---|
| Part I: Multiple Choice | 12 | | |
| Part II: Written | | | |
| 1 | 3 | | Mr Awad |
| 2 | 3 | | Mr Awad |
| 3 | 3 | | Mr Al-Labadi |
| 4 | 4 | | Mr Al-Labadi |
| 5 | 3 | | Mr Saleh |
| 6 | 3 | | Mr Saleh |
| 7 | 4 | | Mr Sharqawi |
| 8 | 3 | | Mr Sharqawi |
| 9 | 6 | | Mr Alzoubi |

| | |
|---|---|
| **Total** | |
| | **44** |

**Part I: ( 12 points) Multiple Choice Questions (MCQ)**

**Bubble the correct answer on the OMR sheet.**

1. Which one of the following relations **DOES NOT** define $y$ as a function of $x$?

   [✓ (a)] $\sqrt{x^2+y^2}=3$
   (b) $\{(5,10),(3,10),(-3,8),(-5,6)\}$
   (c) $x+y^3=1$
   (d) $|x|-y=5$

2. The solution set of the equation $\left[\frac{1}{3}x\right]=-1$, where $[y]$ is the greatest integer less than or equal to $y$, is equal to:

   [✓ (a)] $[-3,0)$
   (b) $[-6,-3)$
   (c) $[-3,-1)$
   (d) $[-3,-1]$

3. Which one of the following statements is **TRUE**?

   [✓ (a)] $\frac{3x\left(x^4-1\right)}{x^2+1}=3x^3-3x$ is an identity.
   (b) $x=3$ and $x^2=9$ are equivalent equations.
   (c) $\frac{x}{x-3}=\frac{3}{x-3}$ is a conditional equation.
   (d) $2x=3(x-1)-x+3$ is a contradiction.

4. If the circumference of a circle is 10 meters, then the area of the circle is equal to:

   [✓ (a)] $\frac{25}{\pi}$ square meters
   (b) $25\pi$ square meters
   (c) $\frac{\pi}{25}$ square meters
   (d) $50\pi$ square meters

5. The line $2x + 3y + 6 = 0$

   [✓ (a)] has y-intercept $(0, -2)$ and slope $\frac{-2}{3}$.
   (b) is parallel to the line $3x + 2y + 6 = 0$
   (c) is perpendicular to the line $2y = -3x$.
   (d) has y-intercept $(0, -2)$ and x-intercept $(3, 0)$

6. The equation $3x^2 + 4x = 5$ has:

   [✓ (a)] two distinct irrational roots.
   (b) two equal real roots.
   (c) two distinct rational roots.
   (d) two distinct non-real roots.

7. If the point $(a, b)$ lies in quadrant $III$, then the point$(ab, -1)$ lies in quadrant

   [✓ (a)] $IV$
   (b) $III$
   (c) $II$
   (d) $I$

8. The domain of $f(x) = \sqrt{\frac{x^2+1}{3-x}}$ is:

   [✓ (a)] $(-\infty, 3)$
   (b) $(-\infty, 3) \cup (3, \infty)$
   (c) $(-3, -1)$
   (d) $(-3, -1) \cup (1, \infty)$

## Part II: Written Questions

**Provide neat and complete solution to each question.**
**Show necessary steps for full credit.**

1. ( 3 points) Write the complex number $z = \frac{\sqrt[3]{-8}\sqrt{-4}+1}{1-i^{13}}$, where $i = \sqrt{-1}$, in standard form.

$$z = \frac{(-2)(2i)\ +1}{1-i} \quad \text{--- 1 point}$$

$$= \frac{(1-4i)(1+i)}{(1-i)\ (1+i)} \quad \text{--- 1 point}$$

$$= \frac{1-3i+4}{1+1}$$

$$= \frac{5}{2} - \frac{3}{2}i \quad \text{--- 1 point}$$

2. ( 3 points) Solve $4x^2 - 3x + 9 = x - 6$ by **completing the square.**

$$\left.\begin{aligned} &\Rightarrow 4x^2 - 4x = -15 \\ &\Rightarrow x^2 - x = -\frac{15}{4} \\ &\Rightarrow \left(x-\frac{1}{2}\right)^2 = -\frac{15}{4} + \frac{1}{4} = -\frac{14}{4} \end{aligned}\right\} \text{--- 2 point}$$

$$\left.\begin{aligned} &\Rightarrow x - 1/2 = \pm i \frac{\sqrt{14}}{2} \\ &\Rightarrow x = \frac{1}{2} \pm i \frac{\sqrt{14}}{2}. \end{aligned}\right\} \text{--- 1 point}$$

3. ( 4 points) Solve the inequality $\frac{x-4}{x-1} \leq -x$, then write the solution set in interval notation.

$$\left.\begin{aligned} &\Rightarrow \frac{x-4}{x-1} + x \leqslant 0 \\ &\Rightarrow \frac{x-4+x^2-x}{x-1} \leqslant 0 \\ &\Rightarrow \frac{x^2-4}{x-1} \leqslant 0 \\ &\Rightarrow \frac{(x+2)(x-2)}{x-1} \leqslant 0 \end{aligned}\right\} \text{ --- 2 points}$$

The sign digram is

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x+2$ | $--$ | | $++$ | | $++$ | | $++$ |
| $x-1$ | $--$ | | $--$ | | $++$ | | $++$ |
| $x-2$ | $--$ | | $--$ | | $--$ | | $++$ |
| | $-$ | $-2$ | $+$ | $1$ | $-$ | $2$ | $+$ |

$\}$ --- 1 point

$\Rightarrow$ The solution set $= (-\infty, -2] \cup (1, 2]$ --- 1 point

4. ( 3 points) Find an equation of a circle that has a diameter with endpoints $(-1, -2)$ and $(7, -2)$. Write your answer in standard form.

The center is at $\left(\frac{-1+7}{2}, \frac{-2-2}{2}\right) = (3, -2)$ --- 1 point

The radius = The distance between $(3, -2)$ and $(-1, -2)$ or $(7, -2)$

$= \sqrt{(3+1)^2 + (-2+2)^2} = \sqrt{(4)^2} = 4$ --- 1 point

$\Rightarrow$ The required equation is

$(x-3)^2 + (y+2)^2 = 16$ . --- 1 point .

5. ( 3 points) Solve the compound inequality:

$$-4x + 5 > 9 \text{ AND } |x - 1| < 3.$$

( Write your answer in interval notation)

$\Rightarrow \quad -4x > 4 \quad \text{AND} \quad -3 < x-1 < 3$ --- 1 point

$\Rightarrow \quad x < -1 \quad \text{AND} \quad -2 < x < 4$ --- 1 point

$\Rightarrow$ The solution set $= (-2, -1)$ --- 1 point

6. ( 3 points) Solve: $(8x+8)^{\frac{2}{3}} - 3\,(x+1)^{\frac{1}{3}} + 1 = 0$

$\Rightarrow 4(x+1)^{2/3} - 3(x+1)^{1/3} - 1 = 0$

$\Rightarrow 4y^2 - 3y - 1 = 0$, where $y = (x+1)^{1/3}$ } --- 1 Point

$\Rightarrow (4y+1)(y-1) = 0$

$\Rightarrow y = -\frac{1}{4}$ or $y = 1$ } --- 1 Point

$\Rightarrow (x+1)^{1/3} = -\frac{1}{4} \Rightarrow x+1 = -\frac{1}{64} \Rightarrow x = -\frac{65}{64}$

OR $(x+1)^{1/3} = 1 \Rightarrow x+1 = 1 \Rightarrow x = 0$

$\Rightarrow$ The solutions are : $-\frac{65}{64}$, $0$. } --- 1 point

7. ( 4 points) If the line through the points $(3k, -1)$ and $(2, k)$ is perpendicular to the line $2x - 5y + 1 = 0$, then find the value of $k$.

The slope of the line through $(3k,-1)$ and $(2,k)$

$= \frac{k+1}{2-3k}$ --- 1 point

The slope of $2x-5y+1=0$ is $\frac{2}{5}$ --- 1 point

$\Rightarrow \quad \frac{k+1}{2-3k} = -\frac{5}{2}$ --- 1 point

$\Rightarrow \quad 2k+2 = -10+15k$

$\Rightarrow \quad 13k = 12 \Rightarrow k = \frac{12}{13}.$ ... 1 point

8. ( 3 points) Solve: $x = 2 + \sqrt{2-x}$

$\Rightarrow \quad x-2 = \sqrt{2-x}$

$\Rightarrow \quad x^2-4x+4 = 2-x$

$\Rightarrow \quad x^2-3x+2 = 0$

--- 1 point

$\Rightarrow \quad (x-1)(x-2) = 0$

$\Rightarrow$ Possible solutions are 1, 2

... 1 point

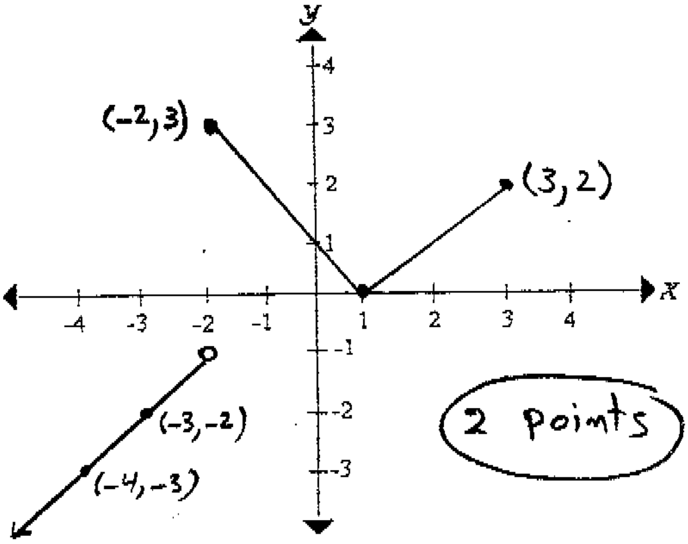
$\Rightarrow$ 2 Checks as a solution, but 1 does not

$\Rightarrow$ 2 is the only solution.

1 point

9. ( 6 points) Given $f(x) = \begin{cases} x+1 & \text{if } x < -2 \\ |x-1| & \text{if } -2 \le x \le 3 \end{cases}$

(a) Sketch the graph of $f$.



2 points

(b) Use Part $(a)$ to find:

i. The domain of $f$ = $(-\infty, 3]$ ... 1 point

ii. The range of $f$ = $(-\infty, -1) \cup [0, 3]$ ... 1 point

iii. The interval(s) over which $f$ is increasing or decreasing.

A. Increasing on: $(-\infty, -1) \cup [1, 3]$ ... 1 point

B. Decreasing on: $[-2, 1]$ ... 1 point