

SOLUTIONS

King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math101-Term072-Quiz-One

Name: _____ ID: _____ Sec.: _____ Serial: _____

Q.1. Find a number δ such that $|f(x) - 3| < 0.02$ whenever $0 < |x + 2| < \delta$, where $f(x) = 2x + 7$

$$|2x + 7 - 3| < 0.02 \text{ whenever } 0 < |x + 2| < \delta$$

$$|2x + 4| < 0.02 \text{ whenever } 0 < |x + 2| < \delta$$

$$2|x + 2| < 0.02 \text{ whenever } 0 < |x + 2| < \delta$$

$$|x + 2| < \frac{0.02}{2} = 0.01 \text{ whenever } 0 < |x + 2| < \delta$$

Choose $\delta = 0.01$ **(4-Points)**

Q2. Let $f(x) = \begin{cases} a + bx, & \text{if } x > 2 \\ 3, & \text{if } x = 2 \\ b - ax^2, & \text{if } x < 2 \end{cases}$ Determine the values of constants a and b so that $f(x)$ is

continuous at $x = 2$

$f(x)$ is continuous at $x = 2 \Rightarrow \lim_{x \rightarrow 2^-} f(x) \text{ exist} \Rightarrow \lim_{x \rightarrow 2^-} f(x) = f(2)$ and $\lim_{x \rightarrow 2^+} f(x) = f(2)$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (b - ax^2) = b - 4a = 3 \dots \dots \dots (1)$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (a + bx) = a + 2b = 3 \dots \dots \dots (2)$$

From equation (1) $b = 3 + 4a$ and substitute it in equation (2) to get

$$a + 2(3 + 4a) = 3 \Rightarrow a + 6 + 8a = 3 \Rightarrow 9a = -3 \Rightarrow a = -\frac{1}{3}$$

$$b = 3 + 4\left(-\frac{1}{3}\right) = 3 - \frac{4}{3} = \frac{5}{3}$$
(6-Points)

Q.3 consider the following graph of the function $y = f(x)$.

Answer the following: **(5+3+2=10-Points)**

- a. $\lim_{x \rightarrow -5^-} f(x) = 7$ b. $\lim_{x \rightarrow -5^+} f(x) = -1$
 b. $\lim_{x \rightarrow 1^-} f(x) = 3$ d. $f(1) = -2$ e. $f(4)$ undefined

f. The discontinuity points are:

When $x = -5, x = 1$ and $x = 4$

g. Which one of the discontinuity points is **removable**? Why?

$x = 4$, because $\lim_{x \rightarrow 4} f(x) = 5$ exist

