

SOLUTIONS

King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics

Math101-Term072-Quiz1

Name: _____

ID: _____

Sec.: _____

Serial: _____

Q.1. Find a number δ such that $|f(x) - 5| < 0.1$ whenever $0 < |x - 1| < \delta$, where $f(x) = 2x + 3$

$$|2x + 3 - 5| < 0.1 \text{ whenever } 0 < |x - 1| < \delta$$

$$|2x - 2| < 0.1 \text{ whenever } 0 < |x - 1| < \delta$$

$$2|x - 1| < 0.1 \text{ whenever } 0 < |x - 1| < \delta$$

$$|x - 1| < \frac{0.1}{2} = 0.05 \text{ whenever } 0 < |x - 1| < \delta$$

Choose $\delta = 0.05$ **(4-Points)**

Q.2. Let $f(x) = \begin{cases} a + bx, & \text{if } x > 2 \\ 3, & \text{if } x = 2 \\ b - ax^2, & \text{if } x < 2 \end{cases}$ Determine the values of constants a and b so that $f(x)$ is

continuous at $x = 2$

$f(x)$ is continuous at $x = 2 \Rightarrow \lim_{x \rightarrow 2^-} f(x) \text{ exist} \Rightarrow \lim_{x \rightarrow 2^-} f(x) = f(2) \text{ and } \lim_{x \rightarrow 2^+} f(x) = f(2)$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (b - ax^2) = b - 4a = 3 \dots \dots \dots (1)$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (a + bx) = a + 2b = 3 \dots \dots \dots (2)$$

From equation (1) $b = 3 + 4a$ and substitute it in equation (2) to get

$$a + 2(3 + 4a) = 3 \Rightarrow a + 6 + 8a = 3 \Rightarrow 9a = -3 \Rightarrow a = -\frac{1}{3}$$

$$b = 3 + 4\left(-\frac{1}{3}\right) = 3 - \frac{4}{3} = \frac{5}{3}$$

(6-Points)

Q.3 consider the following graph of the function $y = f(x)$.

Answer the following: **(5+2+3=10-Points)**

a. $\lim_{x \rightarrow -2^-} f(x) = 4$

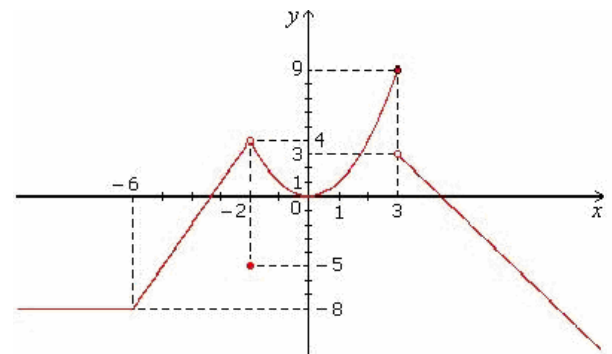
b. $\lim_{x \rightarrow -2^+} f(x) = 4$

c. $\lim_{x \rightarrow 3^-} f(x) = 9$

d. $f(-2) = -5$

e. $f(3) = 9$

f. The discontinuity points are:

When $x = -2$ and $x = 3$ g. Which one of the discontinuity points is **removable**? Why?

$x = -2$, is a removable discontinuity point because $\lim_{x \rightarrow -2} f(x) = 4$ (exists)