SOLUTIONS

King Fahd University of Petroleum & Minerals

Department of Mathematics & Statistics -**Math101-Term072-Quiz4-B**

Name: ID: Sec.: Serial:

Q.1 Find the point on the curve $f(x)=\cosh(x)$ for which the tangent line is perpendicular to the line $y=-x-6$

$f'(x)=\sinh(x)$, the tangent is perpendicular to the line $\Rightarrow m_1.m_2=-1 \Rightarrow m_1=\frac{-1}{m_2}=\frac{-1}{-1}=1$

$\sinh(x)=1 \Rightarrow x=\sinh^{-1}(1)=\ln\left(1+\sqrt{2}\right)$

$y=\cosh\left(\ln\left(1+\sqrt{2}\right)\right)=\sqrt{2}$, so the point is $\left(\ln\left(1+\sqrt{2}\right),\sqrt{2}\right)$ **(2-Points)**

Q2. If a snowball melts so that its surface area decreases at a rate of 1 $cm^2/\min$. find the rate at which the volume decreases when the diameter is 10 cm NOTE: $\left(Volume=\frac{4}{3}\pi r^3,\ Surface\ Area=4\pi r^2\right)$

Let V: be the volume, S: the surface area, X: the diameter (2. radius = 2.r)

$\frac{dS}{dt}=-1cm^2/\min.\ X=2r \Rightarrow r=\frac{X}{2}$ **(1-Point)**

$V=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\left(\frac{X}{2}\right)^3=\frac{4}{3}\pi\frac{X^3}{8}=\frac{1}{6}\pi X^3$

But $S=4\pi r^2=4\pi\left(\frac{X}{2}\right)^2=4\pi\frac{X^2}{4}=\pi X^2$

$\frac{dS}{dt}=2\pi X\frac{dX}{dt} \Rightarrow -1=2\pi(10)\frac{dX}{dt} \Rightarrow\Rightarrow \frac{dX}{dt}=\frac{-1}{20\pi}cm/\min.$ **(1-Point)**

$\frac{dV}{dt}=\frac{1}{2}\pi X^2\frac{dX}{dt}$ **(1-Point)**

$\left.\frac{dV}{dt}\right|_{x=10}=\frac{1}{2}\pi(10)^2\frac{-1}{20\pi}=-\frac{5}{2}cm^3/\min.$ **(1-Point)**

Q3. Use differentials to estimate $\sin\left(61^\circ\right)$

Let

$f(x)=\sin(x),\ x=60^\circ$

$x+\Delta x=x+dx=61^\circ \Rightarrow dx=61^\circ-60^\circ=1^\circ=\frac{\pi}{180}$

$dy=f'(x)dx=\cos(x)dx$

$when\ x=60^\circ,\ dx=\frac{\pi}{180}$ **(4-Points)**

$dy=\cos(60)\frac{\pi}{180}=\frac{1}{2}\frac{\pi}{180}=\frac{\pi}{360}$

$f(x+\Delta x)\approx f(x)+dy=\sin(x)+dy$

$f\left(60^\circ+1^\circ\right)\approx\sin\left(60^\circ\right)+\frac{\pi}{360}=\frac{\sqrt{3}}{2}+\frac{\pi}{360}$