

SOLUTIONS

King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics -**Math101-Term072-Quiz4-A**

Name: _____

ID: _____

Sec.: _____

Serial: _____

Q.1 Find the point on the curve $f(x) = \cosh(x)$ for which the tangent line is parallel to the line $y = x + 5$

$$f'(x) = \sinh(x), \text{ the tangent is parallel to the line } \Rightarrow m_1 = m_2 \Rightarrow m_1 = 1$$

$$\sinh(x) = 1 \Rightarrow x = \sinh^{-1}(1) = \ln(1 + \sqrt{2})$$

$$y = \cosh(\ln(1 + \sqrt{2})) = \sqrt{2}, \text{ so the point is } (\ln(1 + \sqrt{2}), \sqrt{2}) \quad \textbf{(2-Points)}$$

Q2. If a snowball melts so that its surface area decreases at a rate of $1 \text{ cm}^2 / \text{min}$. find the rate at which the diameter decreases when the diameter is 10 cm (NOTE: $\text{Surface Area} = 4\pi r^2$)

Let S: the surface area, X: the diameter (2. radius = 2.r)

$$\frac{dS}{dt} = -1 \text{ cm}^2 / \text{min}. X = 2r \Rightarrow r = \frac{X}{2} \quad \textbf{(1-Point)}$$

$$S = 4\pi r^2 = 4\pi \left(\frac{X}{2}\right)^2 = 4\pi \frac{X^2}{4} = \pi X^2$$

$$\frac{dS}{dt} = 2\pi X \frac{dX}{dt} \quad \textbf{(1-Point)}$$

$$-1 = 2\pi(10) \frac{dX}{dt} \quad \textbf{(1-Point)}$$

$$\Rightarrow \frac{dX}{dt} = \frac{-1}{20\pi} \text{ cm} / \text{min}. \quad \textbf{(1-Point)}$$

Q3. Find the linearization $L(x)$ of the function $f(x) = \sqrt[3]{x}$, $a = -8$, and use it to approximate $\sqrt[3]{-9}$

$$f(x) = \sqrt[3]{x} \Rightarrow f(a) = f(-8) = -2$$

$$f(x) = x^{\frac{1}{3}} \Rightarrow f'(x) = \frac{1}{3}x^{-\frac{2}{3}} \Rightarrow f'(a) = f'(-8) = \frac{1}{3}(-8)^{-\frac{2}{3}} = \frac{1}{3(-8)^{\frac{2}{3}}} = \frac{1}{3\sqrt[3]{(-8)^2}} = \frac{1}{12} \quad \textbf{(1-Point)}$$

$$L(x) = f(a) + f'(a)(x - a) = -2 + \frac{1}{12}(x + 8) = -\frac{4}{3} + \frac{1}{12}x \quad \textbf{(2-Point)}$$

$$\sqrt[3]{x} \approx L(x)$$

$$\sqrt[3]{-9} \approx L(-9) = -\frac{4}{3} + \frac{1}{12}(-9) = -\frac{25}{12} = -2.083 \quad \textbf{(1-Point)}$$