King Fahd University of Petroleum & Minerals

Department of Mathematics & Statistics -**Math101-Term072-Quiz3-B-** SOLUTIONS

Q.1 If $f'(1)=5, g'(1)=-3, f(1)=6, g(1)=-4$, then find $\lim_{h\to 0}\left(\dfrac{\dfrac{f(1+h)}{g(1+h)}-\dfrac{f(1)}{g(1)}}{h}\right)$ if exists.

$$\lim_{h\to 0}\left(\frac{\frac{f(1+h)}{g(1+h)}-\frac{f(1)}{g(1)}}{h}\right)=\left(\frac{f}{g}\right)'(1)\ \textbf{(3-Pts)}=\frac{g(1)f'(1)-f(1).g'(1)}{(g(1))^2}=\frac{(-4).(5)-(6).(-3)}{(-4)^2}\ \textbf{(2-Pts)}$$

$$=\frac{-20+18}{16}=-\frac{2}{16}=-\frac{1}{8}\ \textbf{(1-Point)}$$

Q2. A particle moves on a vertical line so that its coordinate at time t is $S=t^3-12t+3,\ t\geq 0$ where t in seconds, and S in meters, then

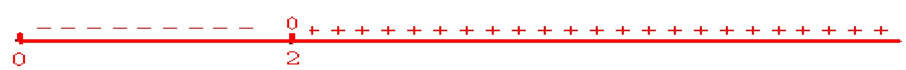
a. Find the distance moved when the velocity is $36\ m/s$

$v(t)=S'(t)=3t^2-12.$ $Set\ v(t)=36\Rightarrow 3t^2-12=36\Rightarrow 3t^2=48\Rightarrow t^2=16\Rightarrow t=+\sqrt{16}=4$ **(2-Pts)**

The distance = $S(4)=(4)^3-12(4)+3=19m$ **(1-Point)**

b. When is the particle moving upward and when it is moving downward?

$Set\ v(t)=0\Rightarrow 3t^2-12=0\Rightarrow 3t^2=12\Rightarrow t^2=4\Rightarrow t=+\sqrt{4}=2$ **(1-Point)**



It is moving upward when $t>2\ or\ (2,\infty)$, **and downward when** $0\leq t<2\ or\ [0,2)$ **(2-Points)**

Q3. Find the points on the curve $y=\dfrac{\cos(x)}{2+\sin(x)}$ at which the tangent is horizontal.

$$y'=\frac{(2+\sin(x))(-\sin(x))-\cos(x)(\cos(x))}{(2+\sin(x))^2}=\frac{-2\sin(x)-\sin^2(x)-\cos^2(x)}{(2+\sin(x))^2}=\frac{-2\sin(x)-1}{(2+\sin(x))^2}\ \textbf{(2-Pts)}$$

Set $y'=0\Rightarrow\dfrac{-2\sin(x)-1}{(2+\sin(x))^2}=0\Rightarrow -2\sin(x)-1=0\Rightarrow\sin(x)=-\dfrac{1}{2}$

$\Rightarrow x=\dfrac{7\pi}{6}+2n\pi\ \ or\ \ x=\dfrac{11\pi}{6}+2n\pi$ **Where** n **is an integer. (2-Points)**

The points are: $x=\dfrac{7\pi}{6}+2n\pi\Rightarrow y=\dfrac{-\sqrt{3}/2}{2+\left(-1/2\right)}=\dfrac{-\sqrt{3}/2}{3/2}=\dfrac{-\sqrt{3}}{3}\Rightarrow\left(\dfrac{7\pi}{6}+2n\pi,\dfrac{-\sqrt{3}}{3}\right)$ **(2-Pts)**

$x=\dfrac{11\pi}{6}+2n\pi\Rightarrow y=\dfrac{\sqrt{3}/2}{2+\left(-1/2\right)}=\dfrac{\sqrt{3}/2}{3/2}=\dfrac{\sqrt{3}}{3}\Rightarrow\left(\dfrac{11\pi}{6}+2n\pi,\dfrac{\sqrt{3}}{3}\right)$ **(2-Points)**