

SOLUTIONS

King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics -**Math101-Term072-Quiz3-A**

Name: _____

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Q.1 If $f'(1) = 5, g'(1) = -3, f(1) = 6, g(1) = -4$, then find $\lim_{h \rightarrow 0} \left(\frac{f(1+h)g(1+h) - f(1)g(1)}{h} \right)$ if exists.

$$\lim_{h \rightarrow 0} \left(\frac{f(1+h)g(1+h) - f(1)g(1)}{h} \right) = (f \cdot g)'(1) \quad \text{(3-Points)}$$

$$= f(1) \cdot g'(1) + g(1) \cdot f'(1) = (6)(-3) + (-4)(5) \quad \text{(2-Points)}$$

$$= -18 - 20 = -38 \quad \text{(1-Point)}$$

Q.2. A particle moves on a vertical line so that its coordinate at time t is $S = t^3 - 12t + 3, t \geq 0$ where t in seconds, and S in meters, then

a. Find the distance moved when the velocity is 15 m/s

$$v(t) = S'(t) = 3t^2 - 12. \text{ Set } v(t) = 15 \Rightarrow 3t^2 - 12 = 15 \Rightarrow 3t^2 = 27 \Rightarrow t^2 = 9 \Rightarrow t = +\sqrt{9} = 3 \quad \text{(2-Pts)}$$

The distance $= S(3) = (3)^3 - 12(3) + 3 = -6 \text{ m}$ **(1-Point)**

b. When is the particle moving upward and when it is moving downward?

$$\text{Set } v(t) = 0 \Rightarrow 3t^2 - 12 = 0 \Rightarrow 3t^2 = 12 \Rightarrow t^2 = 4 \Rightarrow t = +\sqrt{4} = 2 \quad \text{(1-Point)}$$



It is moving upward when $t > 2$ or $(2, \infty)$, and downward when $0 \leq t < 2$ or $[0, 2)$ **(2-Points)**

Q3. Find the points on the curve $y = \frac{\cos(x)}{2 + \sin(x)}$ at which the tangent is horizontal.

$$y' = \frac{(2 + \sin(x))(-\sin(x)) - \cos(x)(\cos(x))}{(2 + \sin(x))^2} = \frac{-2\sin(x) - \sin^2(x) - \cos^2(x)}{(2 + \sin(x))^2} = \frac{-2\sin(x) - 1}{(2 + \sin(x))^2} \quad \text{(2-Pts)}$$

$$\text{Set } y' = 0 \Rightarrow \frac{-2\sin(x) - 1}{(2 + \sin(x))^2} = 0 \Rightarrow -2\sin(x) - 1 = 0 \Rightarrow \sin(x) = -\frac{1}{2}$$

$$\Rightarrow x = \frac{7\pi}{6} + 2n\pi \text{ or } x = \frac{11\pi}{6} + 2n\pi \quad \text{Where } n \text{ is an integer. (2-Points)}$$

$$\text{The points are: } x = \frac{7\pi}{6} + 2n\pi \Rightarrow y = \frac{-\sqrt{3}/2}{2 + (-1/2)} = \frac{-\sqrt{3}/2}{3/2} = -\frac{\sqrt{3}}{3} \Rightarrow \left(\frac{7\pi}{6} + 2n\pi, -\frac{\sqrt{3}}{3} \right) \quad \text{(2-Pts)}$$

$$x = \frac{11\pi}{6} + 2n\pi \Rightarrow y = \frac{\sqrt{3}/2}{2 + (-1/2)} = \frac{\sqrt{3}/2}{3/2} = \frac{\sqrt{3}}{3} \Rightarrow \left(\frac{11\pi}{6} + 2n\pi, \frac{\sqrt{3}}{3} \right) \quad \text{(2-Points)}$$