SOLUTIONS

King Fahd University of Petroleum & Minerals

Department of Mathematics & Statistics -**Math101-Term072-Quiz2**

Name: ID: Sec.: Serial:

Q.1 Let $f(x)=\begin{cases} x^3+x^2-1 & if \quad x\le -2 \\ -4x^2-8x & if \quad x>-2 \end{cases}$ find $f'(x)$ (**Use derivative rules**)

$f'(x)=\begin{cases} 3x^2+2x & if \quad x<-2 \\ -8x-8 & if \quad x>-2 \end{cases}$ **(4 Points)**

At $x=-2$: $f(x)$ is not continuous because

$\lim_{x\to -2^-} f(x)=-5\ne \lim_{x\to -2^+} f(x)=0$ **(2 Points)**

$f'(-2)$ does not exist **(2 Points)**

Q2. Given that f is differentiable at c, let g be defined by: $g(x)=\begin{cases} f(x) & if \quad x\le c \\ f'(c)(x-c)+f(c) & if \quad x>c \end{cases}$

use the <u>derivative definition</u> to find $g'(c)$ (**Use limits only**)

I. $g'_-(c)=\lim_{h\to 0^-}\frac{g(c+h)-g(c)}{h}=\lim_{h\to 0^-}\frac{f(c+h)-f(c)}{h}=f'_-(c)=f'(c)$

Because $f(x)$ is differentiable at $x=c$, then $f'_-(c)=f'_+(c)=f'(c)$ **(2 Points)**

II. $g'_+(c)=\lim_{h\to 0^+}\frac{g(c+h)-g(c)}{h}=\lim_{h\to 0^+}\frac{f'(c)(c+h-c)+f(c)-f(c)}{h}$ **(2 Points)**

$=\lim_{h\to 0^+}\frac{h f'(c)}{h}=\lim_{h\to 0^+} f'(c)=f'(c)$

Because $g'_-(c)=g'_+(c)=g'(c)=f'(c)$ **(2 Points)**

Q.3 Given that $f'(2)=5$ find $\lim_{t\to 0}\frac{f(2+t^2-3t)-f(2)}{t}$

$\lim_{t\to 0}\frac{f(2+t^2-3t)-f(2)}{t}=\lim_{t\to 0}\frac{f(2+t(t-3))-f(2)}{t}$ **(2 Points)**

$=\lim_{t\to 0}\frac{f(2+t(t-3))-f(2)}{t}\cdot\frac{(t-3)}{(t-3)}$ **(2 Points)**

$=\lim_{t\to 0}\frac{f(2+t(t-3))-f(2)}{t(t-3)}\cdot\lim_{t\to 0}(t-3)\Rightarrow \lim_{t\to 0}\frac{f(2+h)-f(2)}{h}\cdot\lim_{t\to 0}(t-3)=f'(2)(-3)=-15$

$Let \ h=t(t-3)\ , \quad As\ t\to 0,\ h\to 0$ **(2 Points)**