

SOLUTIONS

King Fahd University of Petroleum & Minerals
Department of Mathematical Science- MATH-102-Term051-Quiz #6

Name: _____

ID: _____

Serial: _____

Q1. Determine whether the following series converges and if so find its sum.

$$\frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{(k+1)(k+2)} + \dots$$

Let $S_1 = \frac{1}{2.3}$, $S_2 = \frac{1}{2.3} + \frac{1}{3.4}$, ..., $S_n = \frac{1}{2.3} + \frac{1}{3.4} + \dots + \frac{1}{(n+1)(n+2)}$

The n -th partial sum: $S_n = \sum_{k=1}^n \left(\frac{1}{(k+1)(k+2)} \right)$

$$\frac{1}{(k+1)(k+2)} = \frac{A}{k+1} + \frac{B}{k+2}$$

$$= \frac{A(k+2) + B(k+1)}{(k+1)(k+2)} \Rightarrow A(k+2) + B(k+1) = 1$$

$$\Rightarrow A=1 \quad \text{and} \quad B=-1$$

$$S_n = \sum_{k=1}^n \left(\frac{1}{k+1} - \frac{1}{k+2} \right)$$

$$= \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{5} \right) + \dots + \left(\frac{1}{n+1} - \frac{1}{n+2} \right)$$

$$= \frac{1}{2} - \frac{1}{n+2}$$

$$S = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \left(\frac{1}{2} - \frac{1}{n+2} \right) = \frac{1}{2}, \text{ Converges.}$$

Q2. Use any method to determine whether the series converges

a. $\sum_{k=1}^{\infty} \left(1 + \frac{2}{k} \right)^{-2k}$

$$\lim_{k \rightarrow \infty} a_k = \lim_{k \rightarrow \infty} \left(1 + \frac{2}{k} \right)^{-2k} = \left(\lim_{k \rightarrow \infty} \left(1 + \frac{2}{k} \right)^k \right)^{-2}$$

$$= (e^2)^{-2} = e^{-4} \neq 0$$

$$\therefore \sum_{k=1}^{\infty} \left(1 + \frac{2}{k} \right)^{-2k} \text{ diverges.}$$

b. $\sum_{k=0}^{\infty} \frac{(k!)^2}{(2k)!}$

$$\rho = \lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} = \lim_{k \rightarrow \infty} \frac{((k+1)!)^2}{(2(k+1))!} \cdot \frac{(2k)!}{(k!)^2}$$

$$= \lim_{k \rightarrow \infty} \frac{((k+1)(k!))^2 (2k)!}{(2k+2)! (k!)^2} = \lim_{k \rightarrow \infty} \frac{(k+1)^2 (k!)^2 (2k)!}{(2k+2)(2k+1)(2k)! (k!)^2}$$

$$= \lim_{k \rightarrow \infty} \frac{(k+1)^2}{(2k+2)(2k+1)} = \frac{1}{4} < 1$$

$$\therefore \text{by ratio test, } \sum_{k=0}^{\infty} \frac{(k!)^2}{(2k)!} \text{ Converges.}$$