SOLUTIONS

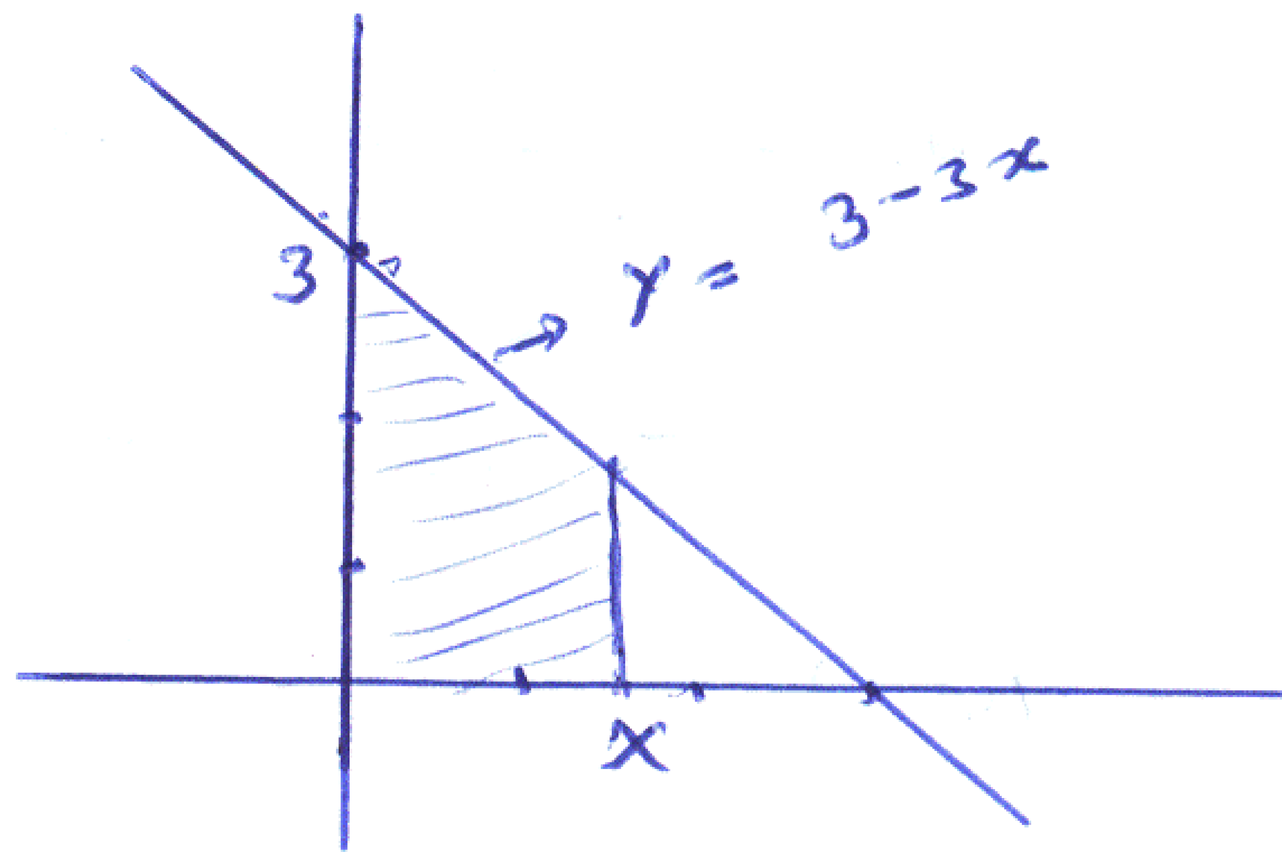
King Fahd University of Petroleum & Minerals
Department of Mathematical Science
MATH-102-Term051-Quiz #1

Name: ID: Serial:

Question One (3-Points)

Suppose that $f(x)=3-3x$; $[a,x]=[0,x]$ find (where $x<3$)

a. The area function $A(x)$



$A(x)$ = Area of tripozoid.

$= \frac{1}{2}\left(3+(3-3x)\right)(x)$

$= \frac{1}{2}x(6-3x)$

2

b. What is the area if $[a,x]=[0,1]$

The area $= A(1) = \frac{1}{2}(1)(6-3) = \frac{3}{2}$. 1

Question Two (3-Points)

Evaluate: $\displaystyle\int \frac{dx}{x\sqrt{4x^2-5}} = \int \frac{dx}{x\sqrt{4(x^2-\frac{5}{4})}} = \frac{1}{2}\int \frac{dx}{x\sqrt{x^2-5/4}}$

put $\frac{x}{\sqrt{5}/2} = u \Rightarrow \frac{2}{\sqrt{5}}x = u \Rightarrow \frac{2}{\sqrt{5}}dx = du \Rightarrow dx = \frac{\sqrt{5}}{2}.du$

$$I = \frac{1}{2}\int \frac{du}{\frac{\sqrt{5}}{2}.u\sqrt{\frac{5}{4}u^2-\frac{5}{4}}}\cdot\frac{\sqrt{5}}{2}$$

2

$$= \frac{1}{2}\int \frac{du}{\frac{\sqrt{5}}{2}u\sqrt{u^2-1}} = \frac{1}{2}\cdot\frac{2}{\sqrt{5}}\sec^{-1}|u| + C$$ 1 point for c

$$= \frac{1}{\sqrt{5}}\sec^{-1}\left|\frac{2x}{\sqrt{5}}\right| + C.$$

Question Three (4-Points)

Find the net signed area using x_k^* the right end point if $f(x)=x^3-1$, $a=0, b=1$.

$$\Delta x = \frac{b-a}{n} = \frac{1}{n}, \quad x_k^* = x_k = 0 + \frac{1}{n}\cdot k = \frac{k}{n}$$

$$A_n = \sum_{k=1}^{n} f(x_k^*)\,\Delta x = \sum_{k=1}^{n}\left(\frac{k^3}{n^3}-1\right)\left(\frac{1}{n}\right)$$

(2)

$$= \sum_{k=1}^{n}\left(\frac{k^3}{n^4}-\frac{1}{n}\right)$$

The net-signed area $= \lim_{n\to\infty} A_n$ (1)

$$= \lim_{n\to\infty} \sum_{k=1}^{n}\left(\frac{k^3}{n^4}-\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n^4}\sum_{k=1}^{n} k^3 - \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n} 1$$

$$= \frac{1}{4} - 1 = -\frac{3}{4}.$$ (1)