SOLUTIONS

King Fahd University of Petroleum & Minerals

Department of Mathematical Science- MATH-102-Term051-Quiz #4

Name: ID Serial:

Evaluate the following integrals:

6 a. $\int \sin(\ln(x))dx$

Let $u = \ln x \Rightarrow du = \frac{1}{x}dx \Rightarrow dx = x\,du$ } ①

$I = \int \sin(\ln x)\,dx = \int \sin u \cdot x\,du$ but $x = e^u$

$I = \int e^u \sin u\,du$

$u_1 = e^u \quad dv_1 = \sin u\,du$

$du_1 = e^u du \quad v_1 = -\cos u$ $\Rightarrow I = e^u \cos u + \underbrace{\int e^u \cos u\,du}_{I_1}$ } ②

$I_1 = \int e^u \cos u\,du$

$u_2 = e^u \quad dv_2 = \cos u\,du$

$du_2 = e^u du \quad v_2 = \sin u$

$I = -e^u \cos u + e^u \sin u - \underbrace{\int e^u \sin u\,du}_{I}$ } ②

$2I = e^u(\sin u - \cos u) \Rightarrow I = \frac{e^u}{2}(\sin u - \cos u) + C$

$= \frac{x}{2}(\sin(\ln x) - \cos \ln x) + C$ } ①

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4 b. $\int \tan^4(x)\sec(x)dx$ $\tan^2 x + 1 = \sec^2 x$

$I = \int \tan^2 x \cdot \tan^2 x \cdot \sec x\,dx$

$= \int (\sec^2 x - 1)^2 \cdot \sec x\,dx$

$= \int (\sec^4 x - 2\sec^2 x + 1)\sec x\,dx$

$= \int (\sec^5 x - 2\sec^3 x + \sec x)\,dx$ } ① pt

$\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$

$I_1 = \int \sec^5 x\,dx = \frac{1}{4}\sec^3 x \tan x + \frac{3}{4}\int \sec^3 x\,dx$

$= \frac{1}{4}\sec^3 x \tan x + \frac{3}{4}\left(\frac{\sec x \tan x}{2} + \frac{1}{2}\int \sec x\,dx\right)$

$= \frac{1}{4}\sec^3 x \tan x + \frac{3}{8}\sec x \tan x + \frac{3}{8}\ln|\sec x + \tan x| + C_1$

3 $I = \frac{1}{4}\sec^3 x \tan x + \frac{3}{8}\sec x \tan x + \frac{3}{8}\ln|\sec x + \tan x| - 2\left(\frac{\sec x \tan x}{2} + \frac{1}{2}\ln|\sec x + \tan x|\right) + \ln|\sec x + \tan x| + C$

$= \frac{1}{4}\sec^3 x \tan x - \frac{5}{8}\sec x \tan x + \frac{3}{8}\ln|\sec x + \tan x| + C$

(4) c. $\int_{\sqrt{2}}^{2} \frac{\sqrt{2x^2-4}}{x}\,dx$

Let $\sqrt{2}\sec\theta = x \Rightarrow \sqrt{2}\sec\theta\tan\theta\,d\theta = dx$ } (1)

when $x=\sqrt{2} \Rightarrow \sec\theta = 1 \Rightarrow \theta = 0$

$x = 2 \Rightarrow \sec\theta = \frac{2}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$.

$$\int_{\sqrt{2}}^{2} \frac{\sqrt{2x^2-4}}{x}\,dx = \int_{0}^{\pi/4} \frac{\sqrt{4\sec^2\theta - 4}}{\sqrt{2}\sec\theta}\cdot \sqrt{2}\sec\theta\tan\theta\,d\theta$$

} (2)

$$= 2\int_{0}^{\pi/4} \tan^2\theta\,d\theta = 2\int_{0}^{\pi/4} (\sec^2\theta - 1)\,d\theta$$

$$= 2(\tan\theta - \theta)\Big]_{0}^{\pi/4}$$

$$= 2\left(1 - \frac{\pi}{4} - 0\right) = 2 - \frac{\pi}{2}.$$ } (1)

(6) d. $\int \frac{x^2}{x^2-5x+4}\,dx$

$$\begin{array}{r|l} x^2-5x+4 & 1 \\ \hline & x^2 \\ & -x^2 \pm 5x \mp 4 \\ \hline & 5x-4 \end{array}$$ } (1)

$$I = \int \frac{x^2}{x^2-5x+4}\,dx = \int \left(1 + \frac{5x-4}{x^2-5x+4}\right)dx$$

$$\frac{5x-4}{(x^2-5x+4)} = \frac{5x-4}{(x-4)(x-1)} = \frac{A}{x-4} + \frac{B}{x-1} = \frac{A(x-1)+B(x-4)}{(x-4)(x-1)}$$ } (2)

$$\Rightarrow A(x-1) + B(x-4) = 5x-4$$

$x=1 \Rightarrow -3B = 1 \Rightarrow B = \frac{-1}{3}$

$x=4 \Rightarrow 3A = 16 \Rightarrow A = \frac{16}{3}$. } (2)

$$I = \int \left(1 + \frac{16/3}{x-4} - \frac{1/3}{x-1}\right)dx$$

$$= x + \frac{16}{3}\ln|x-4| - \frac{1}{3}\ln|x-1| + C$$ } (1)