

*** SOLUTIONS ***

King Fahd University of Petroleum & Minerals

Department of Mathematical Science- MATH-102-Term051-Quiz #4

Name:

ID

Serial:

Evaluate the following integrals:

6 a. $\int \sin(\ln(x)) dx$

Let $u = \ln x \Rightarrow du = \frac{1}{x} dx \Rightarrow dx = x du$ } ①

$I = \int \sin(\ln x) dx = \int \sin u \cdot x du$ but $x = e^u$

$I = \int e^u \sin u du$

$u_1 = e^u \quad dv_1 = \sin u du$

$du_1 = e^u du$

$v_1 = -\cos u$

$\Rightarrow I = e^u \cos u + \int e^u \cos u du$ } ②

$I_1 = \int e^u \cos u du$

$u_2 = e^u \quad dv_2 = \cos u du$

$du_2 = e^u du \quad v_2 = \sin u$

$I = -e^u \cos u + e^u \sin u - \int e^u \sin u du$ } ②

$2I = e^u (\sin u - \cos u) \Rightarrow I = \frac{e^u}{2} (\sin u - \cos u) + C$

$= \frac{x}{2} (\sin(\ln x) - \cos(\ln x)) + C$ } ①

4 b. $\int \tan^4(x) \sec(x) dx$

$\tan^2 x + 1 = \sec^2 x$

$I = \int \tan^2 x \cdot \tan^2 x \cdot \sec x dx$

$= \int (\sec^2 x - 1) \cdot \sec x \cdot \tan^2 x dx = \int (\sec^3 x - \sec x) \cdot \tan^2 x dx$ } ②

Let $u = \sec x \Rightarrow du = \tan^2 x dx$ } ①

$\therefore I = \int (u^3 - u) du = \frac{1}{4} u^4 - \frac{1}{2} u^2 + C$

$= \frac{1}{4} \sec^4 x - \frac{1}{2} \sec^2 x + C$ } ①

4 c. $\int_{\sqrt{2}}^2 \frac{\sqrt{2x^2-4}}{x} dx$

Let $\sqrt{2} \sec \theta = x \Rightarrow \sqrt{2} \sec \theta \tan \theta d\theta = dx$ } ①

When $x = \sqrt{2} \Rightarrow \sec \theta = 1 \Rightarrow \theta = 0$

$x = 2 \Rightarrow \sec \theta = \frac{2}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

$\int_{\sqrt{2}}^2 \frac{\sqrt{2x^2-4}}{x} dx = \int_0^{\pi/4} \frac{\sqrt{4\sec^2\theta-4}}{\sqrt{2}\sec\theta} \cdot \sqrt{2}\sec\theta \tan\theta d\theta$ } ②

$= 2 \int_0^{\pi/4} \tan^2\theta d\theta = 2 \int_0^{\pi/4} (\sec^2\theta - 1) d\theta$

$= 2 (\tan\theta - \theta) \Big|_0^{\pi/4}$

$= 2 (1 - \frac{\pi}{4} - 0) = 2 - \frac{\pi}{2}$ } ①

6 d. $\int \frac{x^2}{x^2-5x+4} dx$

$\frac{x^2-5x+4}{x^2-5x+4} \sqrt{\frac{x^2}{7x^2-5x+4}}$ } ①

$I = \int \frac{x^2}{x^2-5x+4} dx = \int \left(1 + \frac{5x-4}{x^2-5x+4} \right) dx$

$\frac{5x-4}{(x^2-5x+4)} = \frac{5x-4}{(x-4)(x-1)} = \frac{A}{x-4} + \frac{B}{x-1} = \frac{A(x-1) + B(x-4)}{(x-4)(x-1)}$ } ②

$\Rightarrow A(x-1) + B(x-4) = 5x-4$

$x=1 \Rightarrow -3B = 1 \Rightarrow B = -\frac{1}{3}$ } ②

$x=4 \Rightarrow 3A = 16 \Rightarrow A = \frac{16}{3}$

$I = \int \left(1 + \frac{16/3}{x-4} - \frac{1/3}{x-1} \right) dx$ } ①

$= x + \frac{16}{3} \ln|x-4| - \frac{1}{3} \ln|x-1| + C$