SOLUTIONS

King Fahd University of Petroleum & Minerals
Department of Mathematical Science- MATH-102-Term051-Quiz #3

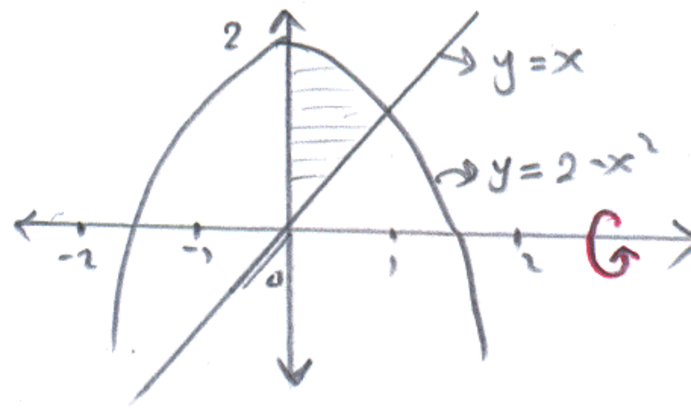
Name: ID: Serial:

Note: (Do not evaluate the integrals)

Question One (6-Points)

Find the volume of the solid that results when the region enclosed by the curves is revolved about the X-axis $y=2-x^2,\ y=x,\ x=0,\ y=0$ using

a. Washers method



$2-x^2=x \Rightarrow x^2+x-2=0 \Rightarrow (x+2)(x-1)=0 \Rightarrow x=-2,\ x=1$ (ignor it)

$V=\pi\int_0^1[(2-x^2)^2-(x)^2]\,dx$

$=\pi\int_0^1(4-5x^2+x^4)\,dx$

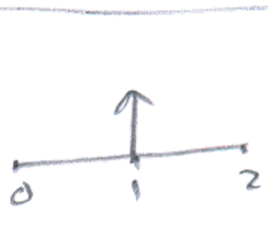
$=\frac{38}{15}\pi$ (3)

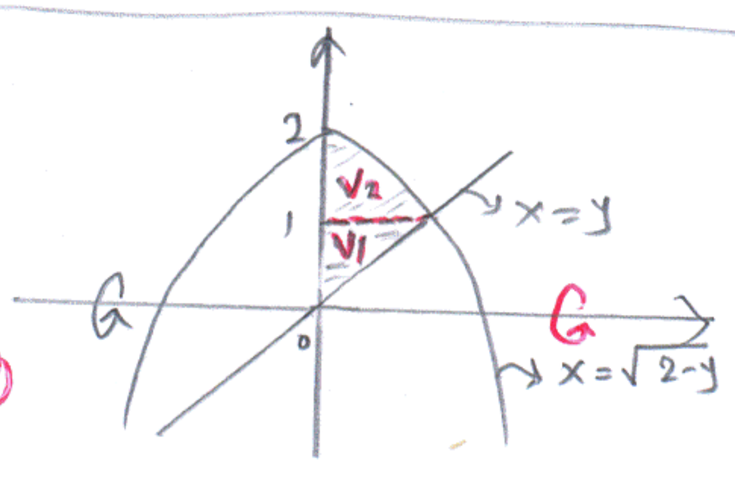
b. Cylindrical method

$x=y,\ x=\pm\sqrt{2-y},\ x=0,\ y=0$

$y=\pm\sqrt{2-y} \Rightarrow y^2+y-2=0 \Rightarrow y=-2,1$ (ignor it)

$\pm\sqrt{2-y}=0 \Rightarrow y=2$ and $y=0$





$V=V_1+V_2=\int_0^1 2\pi y(y)\,dy+\int_1^2 2\pi y\sqrt{2-y}\,dy$ (3)

$=\frac{2\pi}{3}+\frac{28}{15}\pi=\frac{38\pi}{15}$.

Question Two (2-Points)

Find the arc length of $y=\cosh(x)$ between $x=0,\ x=\ln(2)$

$y'=\sinh(x) \Rightarrow 1+(y')^2=1+\sinh^2x=\cosh^2x$

$L=\int_0^{\ln 2}\sqrt{1+(f'(x))^2}\,dx=\int_0^{\ln 2}\sqrt{\cosh^2x}\,dx=\int_0^{\ln 2}\cosh x\,dx$ (2)

$=\sinh(x)\Big|_0^{\ln 2}=\sinh(\ln 2)-\sinh(0)$

$=\frac{3}{4}-0=3/4$

Question Three (2-Points)

Find the area of the surface generated by revolving the curve of $y=\sqrt{4-x^2},\ -1\le x\le 1$ about the X-axis

$y'=\frac{-2x}{2\sqrt{4-x^2}}=\frac{-x}{\sqrt{4-x^2}} \Rightarrow 1+(y')^2=1+\frac{x^2}{4-x^2}=\frac{4-x^2+x^2}{4-x^2}=$

$=\frac{4}{4-x^2}$

$S=\int_{-1}^{1}2\pi\cdot\sqrt{4-x^2}\cdot\sqrt{\frac{4}{4-x^2}}\,dx$ (2)

$=2\pi\int_{-1}^{1}\sqrt{4-x^2}\cdot\frac{2}{\sqrt{4-x^2}}\,dx=4\pi(1+1)=8\pi$