SOLUTIONS

King Fahd University of Petroleum & Minerals
Department of Mathematical Science
MATH-102-Term051-Quiz #2

Name: ID: Serial:

Question One (2-Points)

a. Express the limit as a definite integral(Do not evaluate):

$$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} x_k^* \sin\left(\pi x_k^*\right)\Delta x_k \,, a=0, b=\frac{\pi}{2}$$

$$= \int_0^{\pi/2} x \sin \pi x \, dx \quad \} \ (2)$$

Question Two (4-Points)

Find the average value of the function $f(x)=\dfrac{1}{4+9x^2}$, over the interval $\left[0, \frac{2}{\sqrt{3}}\right]$

$$f_{ave.} = \frac{\int_0^{2/\sqrt{3}} f(x)\,dx}{\frac{2}{\sqrt{3}} - 0}$$

$$\int_0^{2/\sqrt{3}} \frac{1}{4+9x^2}\,dx = \int_0^{2/\sqrt{3}} \frac{1}{9(x^2+\frac{4}{9})}\,dx = \frac{1}{9}\cdot\frac{1}{2/3}\tan^{-1}\left(\frac{x}{2/3}\right)\Big|_0^{2/\sqrt{3}}$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)\Big|_0^{2/\sqrt{3}} = \frac{1}{6}\left(\tan^{-1}\sqrt{3} - \tan^{-1}(0)\right)$$

$$= \frac{1}{6}\left(\frac{\pi}{3} - 0\right) = \frac{\pi}{18}. \quad \} \ (3)$$

$$f_{ave.} = \frac{\pi/18}{\frac{2}{\sqrt{3}}} = \frac{\pi}{18}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}\,\pi}{36}. \quad \} \ (1)$$

Question Three (4-Points)

Let $f(x)=\int_{2x}^{x^2+1} \dfrac{\cos t}{t^2+1}\,dt$ find if possible:

a. $f'(x)$

b. The values of x for which $f(x)=0$

[a] $f'(x) = \dfrac{\cos(x^2+1)}{(x^2+1)^2+1}\cdot 2x - \dfrac{\cos 2x}{(2x)^2+1}\cdot(2)$ } (2) points

[b] $f(x)=0$ when $2x = x^2+1$ } (1)

$\Rightarrow x^2 - 2x + 1 = 0$

$(x-1)^2 = 0 \Rightarrow x-1=0 \Rightarrow x=1$ } (1)