King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Semester II, 2005-2006 (052)
MATH 101 – Exam 3

NAME:________________________________ ID:_______________Section: ___

Part 1: Multiple Choice Questions (1 hour)

1. The function $f(x) = \sqrt{2}x - 2\sin x$ defined on the interval $[0, \pi]$ is:

   A) Decreasing in $[0, \frac{\pi}{4}]$ and increasing in $[\frac{\pi}{4}, \pi]$.

   B) Increasing in $[0, \frac{\pi}{4}]$ and decreasing in $[\frac{\pi}{4}, \pi]$.

   C) Decreasing in $[0, \frac{\pi}{4}]$ and $[\frac{\pi}{2}, \pi]$ and increasing in $[\frac{\pi}{4}, \frac{\pi}{2}]$.

   D) Increasing in $[0, \frac{\pi}{4}]$ and $[\frac{\pi}{2}, \pi]$ and decreasing in $[\frac{\pi}{4}, \frac{\pi}{2}]$.

   E) Increasing in $[0, \frac{\pi}{2}]$ and decreasing in $[\frac{\pi}{2}, \pi]$.

2. The value of $\lim_{x\to 0}(\frac{1}{x} - \frac{1}{e^x - 1})$ is:

   A) $\frac{1}{2}$.

   B) $-\frac{1}{2}$.

   C) Does not exist.

   D) $-\infty$.

   E) $+\infty$.

3. The critical numbers of $f(x) = \sqrt[5]{x^2 - 3x}$ are:

A) $x_0 = \frac{3}{2}$ and all $x \in (0, 3)$.

B) $x_0 = 0$ and all $x > 3$.

C) $x_0 = \frac{3}{2}$ and all $x \neq 0$ and $x \neq 3$.

D) $x_0 = \frac{3}{2}$, $x_1 = 0$, and $x_2 = 3$.

E) $x_0 = \frac{2}{3}$, $x_1 = 0$, and $x_2 = 3$.

4. Which of the following statement is correct:

A) $2\sin^{-1}\sqrt{x} = \frac{\pi}{2} + \tan^{-1}\frac{x-1}{x+1}$.

B) $\sin^{-1}\sqrt{x} = \frac{\pi}{2} + \frac{1}{2}\tan^{-1}\frac{x-1}{x+1}$.

C) $\tan^{-1}\sqrt{x} = \frac{\pi}{4} + \frac{1}{2}\sin^{-1}\frac{x-1}{x+1}$.

D) $\sin^{-1}\frac{x-1}{x+1} = -\frac{\pi}{2} + 2\tan^{-1}\sqrt{x}$.

E) $\sin^{-1}\frac{x-1}{x+1} = -\frac{\pi}{2} + \tan^{-1}\sqrt{x}$.

5. Let $y = x^3 - 2x^2 + 1$. The value of Δy at x=2 when $\Delta x = 0.1$ is:

A) 0.382

B) 0.391

C) 0.4

D) 0.416

E) 0.441.

6. The value of the limit $\lim_{x \to +\infty} \left(\dfrac{x}{x+1} \right)^x$ is:

A) 1.

B) e.

C) e^{-1}.

D) 0.

E) $+\infty$.

7. The height of a right circular cone is three times its radius. If the radius of the cone is decreasing at a constant rate of 1 $cm/\min$, then the rate at which the volume of the cone is changing, when the height of the cone is 6 cm, is equal to: **Hint:** $V = \frac{\pi}{3} r^2 h$

A) $-\ 8\pi\ cm^3/\min$
B) $-\ 32\pi\ cm^3/\min$
C) $-12\pi\ cm^3/\min$
D) $8\pi\ cm^3/\min$
E) $12\pi\ cm^3/\min$.

8. The critical numbers of the function

$$f(x) = \sin^2(x) - 2\cos(x)$$

are:

A) $\{n\pi \mid n \text{ is odd integer}\}$
B) $\{n\pi \mid n \text{ is an integer}\}$
C) $\{2n\pi \mid n \text{ is an integer}\}$
D) $\{\frac{n\pi}{2} \mid n \text{ is odd integer}\}$
E) $\{\frac{3\pi n}{2} \mid n \text{ is an integer}\}$.

9. A linear approximate value of $\dfrac{1}{\sqrt{27}}$ is equal to:

A) $\dfrac{1}{5} - \dfrac{1}{\sqrt[3]{25^2}}$

B) $\dfrac{26}{125}$

C) $\dfrac{48}{250}$

D) $\dfrac{49}{250}$

E) $\dfrac{51}{250}$.

10. Let $g(x) = f(x^2)$, where f is twice differentiable for all x, $f'(x) > 0$ for all $x \neq 0$, and f is concave downward on $(-\infty, 0)$, and concave upward on $(0, \infty)$. Which of the following statement is correct about the function $g(x)$. **Hint:** *Verify the symmetry of* $g(x)$.

A) The function $g(x)$ has a minimum at $x = 0$ and is concave upwards on $(-\infty, 0)$ and is concave downwards on $(0, \infty)$.

B) The function $g(x)$ has a maximum at $x = 0$ and is concave upwards on $(-\infty, 0)$ and is concave downwards on $(0, \infty)$.

C) The function $g(x)$ has a minimum at $x = 0$ and is concave downwards on $(-\infty, \infty)$.

D) The function $g(x)$ has a minimum at $x = 0$ and is concave upwards on $(-\infty, \infty)$.

E) The function $g(x)$ has a minimum at $x = 0$ and is concave downwards on $(-\infty, 0)$ and is concave upwards on $(0, \infty)$.