King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Semester II, 2005-2006 (052)

MATH 101 – Exam 1

NAME:_______________________________________ ID:_____________________ Section: ______

Part 2: Essay Questions (1 hour)

	Score (out of 10)
Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Total (out of 50)	

Question 1

Use the squeezing theorem to find $\lim_{x \to 0} x^2 e^{\sin(\frac{1}{x})}$.

Question 2

Let $\lim_{x \to 3} f(x) = 0$ and $\lim_{x \to 3} h(x) = 5$. Use these limits and the given graph of the function g to evaluate each of the following limits if it exists. If the limit does not exist, explain why.

a) $\lim_{x \to 3} (f(x) - \frac{h(x)}{3})$

b) $\lim_{x \to 3} \frac{g(x) - 3}{h(x)}$

c) $\lim_{x \to 3^-} (g(x) + h(x))^3$

d) $\lim_{x \to 0^+} \frac{1}{\sqrt{g(x)}}$

e) $\lim_{x \to 3} f(x)g(x)$

Question 3

By using the ε and δ definition, prove that $\lim_{x\to 4} \frac{1}{x-2} = \frac{1}{2}$.

Question 4

Prove that the equation $x^2 + x - \cos x = 0$ has at least two solutions in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Question 5

Suppose that f is a continuous function on the interval $[0,1]$ and $f(0) = f(1)$. Prove (analytically and not geometrically) that there exists $a \in (0, \frac{1}{2})$ such that a and $a + \frac{1}{2}$ have the same image, that is, $f(a) = f(a + \frac{1}{2})$.

(**Hint**: consider the function $g(x) = f(x + \frac{1}{2}) - f(x)$)