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On a Singulaly Perturbed Liénard System with Three Equilibrium Points

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We study a family of singularly perturbed Liénard systemd with three equilibrium points, we show the existence of periodic solutions and find that the bifurcation implies the existence of ducks which have in our case a special form.

We consider the system

$$\begin{cases} \varepsilon x' &= y - x^3/3 + x, \\ y' &= -x^3 + \alpha x. \end{cases} \qquad (U_\alpha)$$

The three equilibrium points are: $a_0(0,0)$, $a_1(\sqrt{\alpha}, f(\sqrt{\alpha}))$, $a_2(-\sqrt{\alpha}, f(-\sqrt{\alpha}))$, where $f(x) = x - x^3/3$.

Theorem 1 *i) For $\alpha \ll 1$, the system (U_α) admits a single periodic trajectory.*
ii) For $\alpha \gg 1$, the system (U_α) does not admit a periodic trajectory.

We give the period of the cycle:

Proposition 1 *The period T of the cycle of (U_α) is given by:*

$$T \approx 2\log 2 + \frac{\alpha - 1}{\alpha} \log \frac{4 - \alpha}{4 - 4\alpha}.$$

We also show that (U_α) can have heteroclinic orbits:

Theorem 2 *When $\alpha \gg 1$, the system (U_α) has two heteroclinic orbits.*

The bifurcation of this cycle implies the existence of special trajectories which are called *ducks*:

Theorem 3 *For $\alpha \approx 1$, $\alpha > 1$, the system (U_α) has ducks trajectories.*

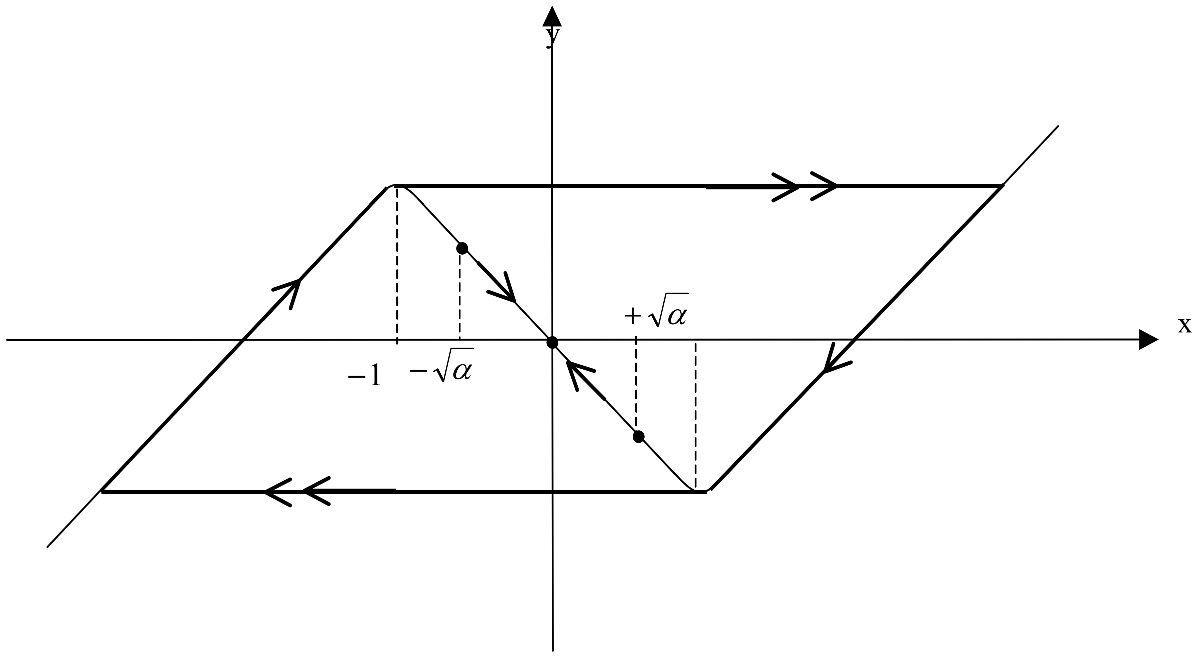

Figure 1: The case $\alpha \ll 1$

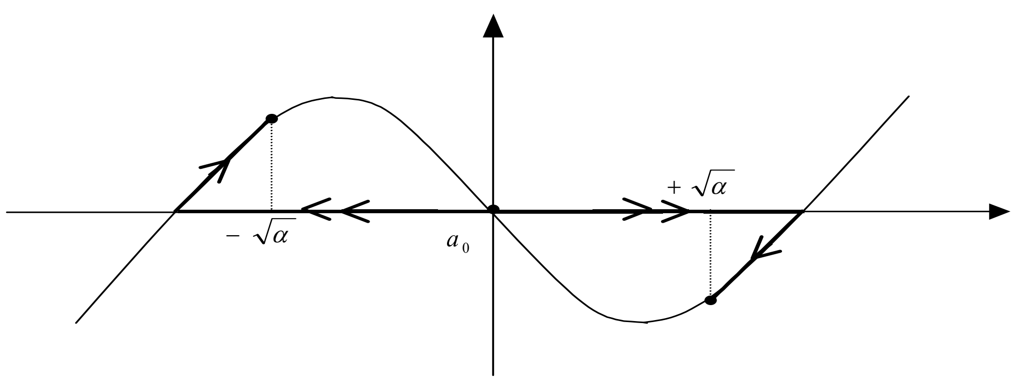

Figure 2: The case $\alpha \gg 1$

These trajectories have a particular form different from that met for the equation of Van der Pol, and this is because of the property of symmetry which the field has, which makes that the symmetrical of the angular point which is the *col* (throat, front part of the neck) of duck, is also an angular point of the cycle duck, we have called it *nuque* (back part of the neck) of duck.

If we denote x_c, x_b, x_q and x_n the abscissae of the cycle duck, there is then the result:

Proposition 2 *If there is a cycle duck of* (U_α) *and* T *is its period, then*

$$T \approx \log(\frac{x_b}{x_c})^2.$$

Proposition 3 *If* α_0 *is the duck's value for which we have the biggest value of* x_c*, then the cycle of* (U_α) *is a duck with head if*

$$\alpha_0 - \alpha \approx \exp\left(-\varepsilon^{-1}\left(\frac{c^2}{2} - \frac{1}{2} - \log c\right)\right) \quad (c = x_c^0).$$

We also show that the system (U_α) can have homoclinic orbits.

Proposition 4 *There exists* $\alpha > 1$, $\alpha \approx 1$ *such that the system* (U_α) *has two homoclinic orbits.*

References

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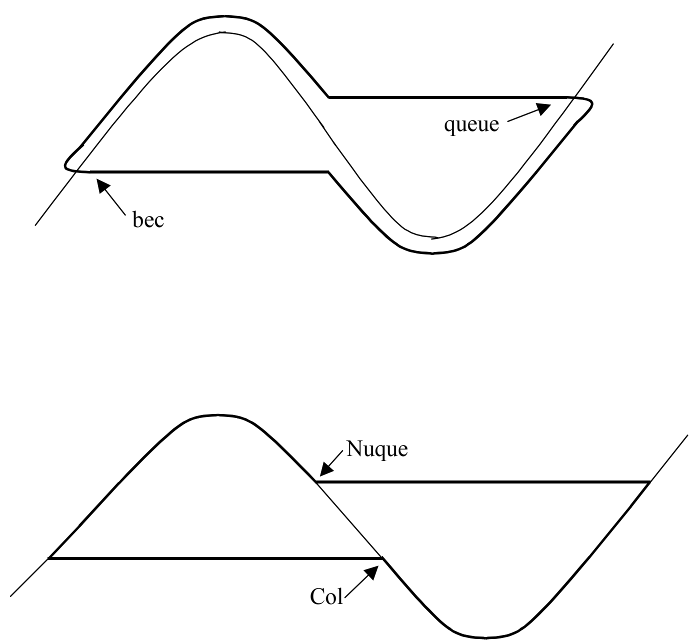

Figure 3: Morphology of ducks

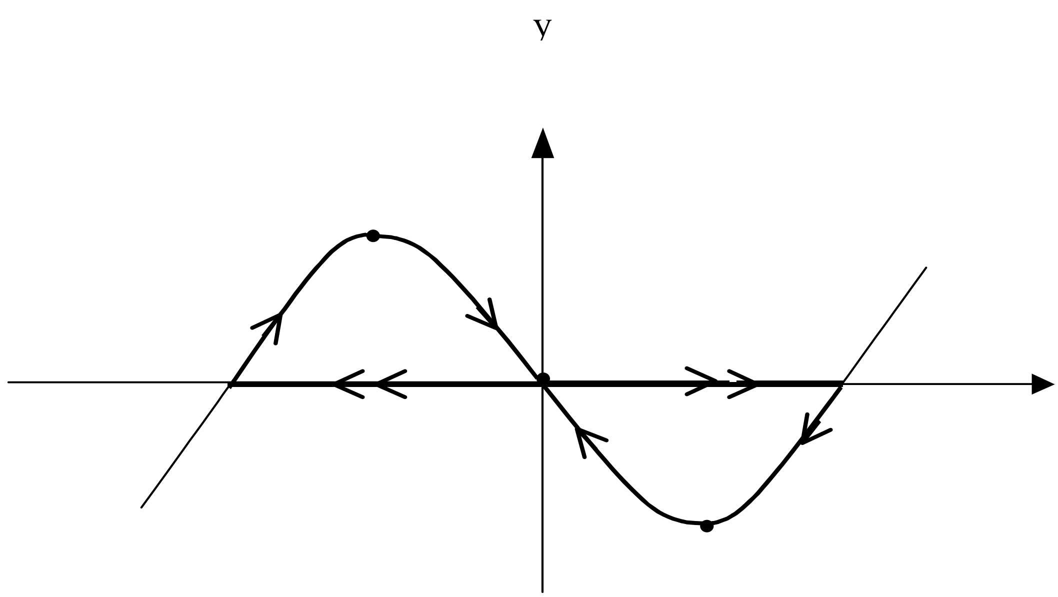

Figure 4: The two homoclinic orbits